## MCDA tools and risk analysis: the decision deck project

Brice Mayag & Olivier Cailloux & Vincent Mousseau Laboratoire de Génie Industriel École Centrale Paris, Chatenay-Malabry, France

Multiple Criteria Decision Aid (MCDA) aims at helping a decision maker (DM) in the representation of his preferences over a set of alternatives, on the basis of several criteria which are often contradictory. In addition to the presence of a DM, MCDA process requires a specification of criteria, alternatives and preferential information given by the DM. Hence many methods have been developed in order to solve MCDA problems (ELECTRE methods, AHP, etc.).

The interconnexion between MCDA and risk analysis has been proved. MCDA methods can be used to solve risk analysis problems such as:

- Computation of a risk scale: it can be done by using MCDA methods as ELECTRE TRI or by MACBETH methodology when the scale is quantitative;
- The evaluation of remediation solutions after an accident.

We present in this paper the Decision Deck (DD) project (http://www.decision-deck.org/) which aims at collaboratively developing Open Source software tools implementing MCDA. These tools constitute an open source platform available for all communities, not only for MCDA community. We show how MCDA methods implemented in Decision Deck can be useful for risk analysis, especially in risk assessment and remediation risk management. Thus, Decision Deck can be interpreted as a bridge between MCDA and risk analysis.

## 1 INTRODUCTION

MultiCriteria Decision Aid aims at helping one or more decision makers (DMs), guided by one or more analysts, to prepare and make a decision where more than one point of view has to be considered. Its objective being not to force a decision at any cost, MCDA can range from a rational structuring of the decision problem to the elaboration of a decision recommendation. In this context, many methods and algorithms have been proposed in the literature. These methods can be schematically divided into two classes of methodologies:

• The outranking methods proposed by the European methodological school. Their objective is to build, using pairwise comparisons, a relation on a set of alternatives called the outranking relation, and to exploit it in order to solve MCDA problems (choice, sorting or ranking). To this category belong the ELECTRE

methods (Figueira, Mousseau, and Roy 2005) and PROMETHE (Brans, Mareschal, and Vincke 1984).

• Methods based on the multi-attribute utility theory proposed by the American methodological school (Keeney and Raiffa 1976). The goal of these methods is to build a numerical representation of the preferences of the DM on the set of alternatives. Methods from this category include MAUT (Dyer 2005), MACBETH (Bana e Costa, Corte, and Vansnick 2005).

The Decision Deck project's objective is to provide an open source software, composed of various modular components, pertaining to the field of Multiple Criteria Decision Analysis (MCDA). It should give a user the possibility to add, modify or simply use existing plugged-in functionalities (plugins). These constituents can either be complete MCDA methods or elements common to a large range of procedures. The typical end-user of the Decision Deck platform is an MCDA researcher, an MCDA consultant or a teacher in an academical institution. The aim of this paper is to present this project to the risk community and to show how MCDA methods implemented in Decision Deck can be useful for risk analysis.

The document is organized as follows. After the basic materials of MCDA given in the next section, we present the Decision Deck projet as a project containing tools for a MCDA process. We end by the use of MCDA technics in risk analysis.

## 2 MCDA CONCEPTS AND METHODS

As clearly stated by B. Roy in his book on Multicriteria Methodology (Roy 1996), Multiple Criteria Decision Aid is an activity that creates models to provide the decision maker (DM) with guidelines with respect to his decision problem. Three basic problems are usually put forward (Meyer 2007; Figueira, Greco, and Ehrgott 2005; Meyer and Roubens 2005):

- the *choice problem* that aims to select a subset of potential alternatives, as restricted as possible, containing the "satisfactory" actions;
- the *sorting problem* that corresponds to the assignment of each alternative into pre-defined categories. These categories correspond to a set M of classes. If M is just a set of labels we talk about a classification problem. If the labels of M can be ordered, we are dealing with an ordered sorting;
- the *ranking problem* that aims at ordering the alternatives by decreasing order of preference. The prescription may be given in terms of a partial or a complete order.

Such a decision analysis process requires in general at least two actors. The first actor is the so-called *decision maker (DM)* which is a person who will take the responsibility for the decision act. He furthermore bears certain values, priorities and preferences related to the particular decision problem. The second actor is an individual, who will facilitate the decision analysis process by investigating thoroughly the underlying problem. He is often called the *analyst*.

The MCDA process begins by the following step:

# 2.1 Elaboration of a set of alternatives, a family of criteria and a performance matrix

The starting point of a MCDA process is a definition of finite set X of p > 1 potential decision objects (also called alternatives). They represent the possible options on which the DM has to make his decision.

Since we are in a multidimensional framework, the alternatives are evaluated on a finite set  $N = \{1, ..., n\}$  of n > 1 attributes. For each attribute  $i \in N$ , we compute a descriptor  $g_i : X \to X_i$  which allows to assess the alternatives on attribute i of N, where  $X_i$  is the set of levels of the associated scale.

Roy (Roy 1996) underlines that the set X has in a first step to be clearly identified and validated by the DM and that the attributes represent all the dimensions that have consequences on the objective of the decision analysis. As we will show later, in particular, two alternatives having the same evaluations on all the selected attributes should be considered as indifferent.

A criterion is the combination of an attribute with supplementary information derived from the DM's preferences. For short, it is a numerical function which represents the attribute together with some of the DM's preferences, as, for example, an order of the different evaluation levels.

From the set of alternatives  $X = \{a_1, \ldots, a_p\}$  and the set of attributes (criteria) N, we compute the matrix  $A = (\alpha_{ij})_{i=1,\ldots,p; j=1,\ldots,n}$ , where an element  $\alpha_{ij}$  is the performance  $g_j(a_i)$  of the alternative  $a_i$  on the attribute j (or criterion  $g_j$ ). The matrix A is given by the following tabular:

	$g_1$	•••	$g_j$	•••	$g_n$
$a_1$	$g_1(a_1)$	• • •	$g_j(a_1)$	•••	$g_n(a_1)$
÷	÷	÷	÷	÷	÷
$a_i$	$g_1(a_i)$		$g_j(a_i)$	•••	$g_n(a_i)$
÷	:	÷	÷	÷	÷
$a_m$	$g_1(a_m)$		$g_j(a_m)$	• • •	$g_n(a_m)$

Table 1: The performance matrix A.

Hence an alternative  $a_i$  can be defined by its performance vector  $(g_1(a_i), \ldots, g_n(a_i))$ .

**Example 1** Mary wants to buy a digital camera for her next trip. To do this, she consults a website where she finds six propositions based on three criteria: battery life of the camera (measured in time unit), price (expressed in euros) and quality (expressed by an adjective). The performance matrix is given by the following table:

Cameras	1 : Battery	2: Price	3 : Quality
a : Nikon	7	150	good
b : Sony	6	155	very good
c : Panasonic	10	160	good
d: Casio	12	175	medium
e : Olympus	10	160	very good
f : Kodak	8	165	good
Using our not	tations, we	have N	$= \{1, 2, 3\},$
$X_1 = [6, 12],$	$X_2 =$	[150, 180]	$X_3 =$

{bad, medium, good, very good} and  $X = X_1 \times X_2 \times X_3$ .

In order to choose the best alternative from the performance matrix, outranking methods construct a outranking relation and methods based on multi-attribute utility theory need an aggregation function.

#### 2.2 Outranking methods

Considering two alternatives x and y of X, an outranking S (Meyer 2007; Figueira, Mousseau, and Roy 2005) between x and y holds (x S y) if it is reasonable to accept, from the DM's point of view, that x is at least as good as y. From this definition, three situations can be distinguished,

- x and y are considered as indifferent if simultaneously x S y and y S x,
- an incomparability situation originates from the complete absence of outranking between x and y (neither x S y nor y S x),
- x is strictly preferred to y if x S y and not y S x.

The construction of the outranking relation is done via pairwise comparisons of the alternatives on each of the criteria. They are based on differences of evaluations which are then compared to preference, indifference and veto thresholds (fixed in accordance with the DM's preferences) in view of elaborating the outranking relation. An additive aggregation of such local relations is then performed via a weighted sum. This requires that to each criterion is associated its importance coefficient (or weight). Finally, this calculation produces a valued outranking relation on the set X which can then be seen as a valued digraph, called the *outranking digraph*. As the outranking relation is not necessarily complete or transitive, this task is in general quite difficult and requires a clear understanding of the semantics linked to the outranking relation.

**Example 2** From the Example 1, let us compute the outranking digraph with the following outranking relation S: an alternative x outranks an alternative y if and only if the number of attributes i such that  $g_i(x) \ge g_i(y)$  is greater than the number of attributes i saying that  $g_i(y) \ge g_i(x)$  i.e.

 $\begin{array}{cccc} x & \tilde{S} & y \Leftrightarrow |\{i \in N | g_i(x) \geq g_i(y)\}| \geq |\{i \in N | g_i(y) \geq g_i(x)\}|. \end{array}$ 

*Hence we obtain as part of S:* 

Nikon S Sony S Panasonic S Nikon

and the following digraph for these three alternatives is represented in Figure 1.

How to exploit this outranking relation in order to select the best camera between Nikon, Sony and



Figure 1: A digraph representing a condorcet paradox

Nikon? The problem here seems not easy because this example leads a condorcet paradox, but it may be solve by using an exploitation procedure.

The second step of an outranking method is to exploit the outranking digraph in order to solve one of the MCDA problematics mentioned above (choice, ranking, ...). Many exploitation procedures have been proposed in the literature such as: the identification of the kernel of the digraph. A kernel of an outranking relation is a subset of alternatives such that

- any alternative which is not in the subset is outranked by at least one alternative of the subset;
- the alternatives of the subset are incomparable.

#### 2.3 Methods based on multi-attribute utility theory

The aim of these methods is to construct a preference relation over X. In practice (see (Chateauneuf, Grabisch, and Rico 2008; Marchant 2003)) one can only ask to the DM pairwise comparisons of alternatives on a finite subset X' of X, X' having a small size. Hence we get a preference relation  $\succeq_{X'}$  on X'. The relation  $\succeq_{X'}$  designates a preferential information of DM on the set of reference subset X'.

The question is then: how to construct a preference relation  $\succeq_X$  on X, so that  $\succeq_X$  is an extension of  $\succeq_{X'}$ ? To this end, people usually suppose that  $\succeq_X$  is representable by an overall utility function  $F \circ U$  such that:

$$x \succeq_X y \Leftrightarrow F(U(x)) \ge F(U(y)) \tag{1}$$

where  $U(x) = (u_1(x_1), \ldots, u_n(x_n)), u_i : X_i \to \mathbb{R}$  is called a utility function, and  $F : \mathbb{R}^n \to \mathbb{R}$  is an aggregation function. There exist many aggregation function in MCDA and each them forms an aggregation model of MCDA. As examples of aggregation functions, we can cite the well known arithmetic mean or weighted sum, the OWA operators and the Choquet integral (Mayag, Grabisch, and Labreuche 2009b; Mayag, Grabisch, and Labreuche 2009a). Let us mention that the utility function  $u_i$  can be determine by the interview of the DM or by using some MCDA methods as MACBETH (Bana e Costa, Corte, and Vansnick 2005) or UTA (Siskos, Grigoroudis, and Matsatsinis 2005).

Usually, we consider a aggregation functions characterized by a parameter vector  $\theta$  (e.g., a weight distribution over the criteria). The parameter vector  $\theta$ can be deduced from the knowledge of  $\succeq_{X'}$ , that is, we determine the possible values of  $\theta$  for which (1) is fulfilled over X'. This step of MCDA process is called the *elicitaion phase* (Mousseau, Figueira, Dias, Silva, and Climaco 2003; Mousseau, Dias, and Figueira 2006).

**Example 3** Let us suppose that, the performance matrix of Mary's problem (see Example 1) with utility functions, given in [0,20] interval, is given by the following table:

Cameras	1 : Battery	2: Price	3 : Quality
a : Nikon	10	18	15
b : Sony	8	15	20
c : Panasonic	14	12	15
d: Casio	16	5	10
e : Olympus	14	12	20
f : Kodak	11	9	15

If we choose as aggregation function, the weighted sum with the same weight for each criterion, then we obtain the following ranking of cameras:

 $\textit{Olympus} \prec \textit{Nikon} \sim \textit{Sony} \prec \textit{Panasonic Kodak} \prec \textit{Casio}.$ 

## 3 DECISION-DECK PROJECT: TOOLS FOR A MCDA PROCESS

The Decision Deck project aims at collaboratively developing Open Source software tools implementing Multiple Criteria Decision Aid (MCDA). Its purpose is to provide effective tools for three types of users:

- practitioners who use MCDA tools to support actual decision makers involved in real world decision problems;
- teachers who present MCDA methods in courses, for didactic purposes;
- researchers who want to test and compare methods or to develop new ones.

From a practical point of view, the Decision Deck project works on developing multiple software resources that are able to interact. Consequently, several complementary efforts focusing on different aspects contribute to Decision Decks various goals.

The project continues and expands the series of activities that have been pursued by the Decision

Deck Community, including at the approval date of the manifesto:  $D^2$  software, XMCDA standard and web services,  $D^3$  software and Diviz software. Some screenshots (see Figures 2, 3 and 4) are introduced in this paper only to show which type of tools we have in Decision Deck project. The readers can access to more details about these screenshots via the website http://www.decision-deck.org/.

## 3.1 $D^2$ software

The Decision Desktop software, or  $D^2$  for short, was the first software to be developed in the Decision Deck project. It is a desktop application, meaning that it is designed to be installed locally (it is not a web application), and uses a database to store application data, thereby enabling a multiple user usage.

One of its usage patterns is that several experts may enter evaluations in a decentralized manner, then these evaluations be analyzed by a coordinator, this analysis being then reviewed by one or several decision-maker. Another interesting features offered by the  $D^2$  software is the common availability of visualisation resources as illustrated in Figure 2.

The following MCDA methods are currently implemented:

- *IRIS*: sorting of alternatives into ordered classes based on an outranking relation;
- *RUBIS*:progressive best choice method based on an outranking relation.
- *VIP*: best choice method based on an additive aggregation model accepting imprecise information on the scaling coefficients,
- *UTA-GMS/GRIP* (Figueira, Greco, and Slowinski 2009; Greco, Mousseau, and Slowinski 2008): ranking alternatives with a set of value functions;

## 3.2 XMCDA standard and web services

XMCDA is a data standard which allows to represent MultiCriteria Decision Analysis (MCDA) data elements in XML according to a clearly defined grammar. The main objective of XMCDA is to allow different MCDA algorithms to interact and be easily callable.

Decision Deck's XMCDA web services are algorithmic components or complete Multicriteria Decision Aid (MCDA) methods which are made available online. The XMCDA web services use the XMCDA data standard to be interoperable. It is a distributed open source computational MCDA resources, like the RUBIS solver written in Python and the KAPPALAB (Choquet integral based MAVT) R library.



Figure 2: A snapshot taken from a  $D^2$  Rubis plugin session shows the performances of the alternatives on a subset of criteria in a column chart style

## 3.3 $D^3$ software

One of the drawback of the  $D^2$  software rich java client software consists in the necessity to program all new MCDA tool in Java. Many researchers have programmed, however, their methods and tools in other systems and languages. The idea of the Distributed Decision-Deck ( $D^3$ ) project is to use the web service technology for distributing these existing ressources to potential  $D^2$  clients. An example of such a ressource is the RUBIS server installed at the University of Luxembourg (see Figure 3). It takes as input a specific performance table and renders as output the RUBIS best choice recommendation. Therefore,  $D^3$  is an open source rich internet application for XMCDA web services management.

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Figure 3: A snapshot taken from a  $D^3$  session

## 3.4 Diviz software

Diviz is a software for *designing, executing and sharing* Multicriteria Decision Aid (MCDA) methods, algorithms and experiments. The goals of diviz are:

- to help researchers to build algorithmic MCDA workflows ( = methods) from elementary MCDA components;
- to help teachers to present MCDA methods and let the students experiment their own creations;
- to help to easily compare results of different methods;
- to allow to easily add new elementary MCDA components;
- to avoid heavy calculations on your local computer by executing the methods on distant servers;
- to ease the dissemination of new MCDA algorithms, methods and experiments.

The main properties of Diviz are:

- available MCDA components in diviz are XM-CDA web services;
- the history of all the past executions is available (ideal for parameter tuning);
- use of XMCDA to make elementary components interoperable;
- use of XMCDA and XSL (eXtensible Stylesheet Language) and CSS (Cascading Style Sheets) for a standardised visualisation of input and output data.



Figure 4: A snapshot of Diviz interface

## 3.5 Research topics and coordination

All the efforts above involve developments on at least one of the following research topics of the Decision Deck project:

- global architecture of MCDA systems;
- implementations and developments of MCDA algorithms;
- data models and management of MCDA objects;
- decision process modeling and management;
- graphical user interface.

In order to coordinate the various activities of the Decision Deck project, it is structured as follows:

- The Decision Deck Consortium: a french non profit association which steers and manages the project along the lines of this manifesto. It is headed by an administration board. The consortium is among other in charge of organizing the workshops of the project every semester.
- The Software Resources Management Groups: they are in charge of the organisation and the management of the developments of the identified initiatives of the Decision Deck project. Each group is coordinated by a clearly identified contact person. These management groups are in charge of organizing the bi-annual developers days of the project.
- The Specifications Committee: under the direction of a coordinator, its role is to maintain and develop the XMCDA standard and to approve and publish suggested evolutions. The coordinator is in charge of organizing the specifications meetings.
- The Communication & Dissemination Committee: under the direction of a coordinator, its role is to develop and maintain the websites of Decision Deck and to manage the communicational aspects of the project.

## 3.6 Future developments

At this step of the Decision Deck project, new versions of XMCDA, XMCDA web services and DI-VIZ are now available on the website http://www. decision-deck.org/. These new versions show that the community still continue to work together and to develop the interesting methods.

As perspectives, we can also mention the  $D^4$  software (Bisdorff and Zam 2009), which is a rationale

concept and implementation of a distributed MCDA application designer. The community investigates the opportunities for using the XMCDA standard within D4 (Bisdorff and Zam 2010). What is the right granularity of practical XMCDA formulated problems? Does XMCDA need UML profiling and stereotyping mechanisms?

We have proposed a french ANR project which aims at studying in details Multiple Criteria Decision Aid (MCDA) processes and to develop generic software tools which can guide all the stakeholders during the decision aid process. These generic tools will be inserted in the Decision Deck project.

## 4 TOWARDS THE USE OF MCDA TECHNICS FOR RISK ANALYSIS

We present two examples to illustrate the applicability of MCDA in the domain of risk. Although chosen simple, these examples are intended only to emphasize that we could forge links between these two areas.

## 4.1 Building a risk evaluation model

The following example is taken in (Cailloux and Mousseau 2011). A group of experts would like to develop a scale permitting to evaluate the level of risk of each territorial zone around a given industrial installation related to a possible hazard. Each zone is to be determined as belonging to one of the three categories {High risk; Medium risk; Low risk}. Each of these categories are associated with specific precautionary measures (e.g. evacuate the population). Building a risk evaluation scale relative to hazards related to industrial installations is typically important in decision processes such as the PPRT (Plan de Prévention des Risques Technologiques) used in France (PPRT 2011).

The set of alternatives is composed zones where the risk will be evaluated. We can consider as criteria the following factors in each zone

- (i) The distance to the hazardous industrial installation, evaluated on a 3 points ordinal scale. 0: less than 500 meters; 1: between 500 and 2000 meters; 2: more than 2 km.
- (ii) Presence or absence school in the zone, a binary assessment. No school is encoded with a 1, presence of school with a 0.
- (iii) Building vulnerability, evaluated on a 5 points ordinal scale.
- (iv) Presence of public environment and technical assets, evaluated on a 5 points ordinal scale.

- (v) Impact on other industrial installations, which could lead to cascaded effects. Evaluated on a 5 points ordinal scale.
- (vi) Proportion of the population who is vulnerable (children under 15 and elderly persons), evaluated in percentage.

This risk evaluation problem can be viewed as a MCDA sorting problem with the three categories {High risk; Medium risk; Low risk}. To solve it, one can use MCDA methods for sorting such as ELEC-TRE TRI (Figueira, Mousseau, and Roy 2005).

The same problem can be solve by computing a cardinal scale u of risk on a set of zones i.e. a numerical scale where the difference of evaluation between two zones has a significant meaning. To do this, we can use the MACBETH (Bana e Costa, Corte, and Vansnick 2005) approach which is a MCDA method dedicated to computation of cardinal scale by collecting preferential information from the interview of the experts. These preferential information are given with the intensity of preferences.

#### 4.2 Choice of prevention measures

Another problem in risk analysis which can be solve by MCDA methods is the problem of the choice of the best prevention measure to elaborate in order to prevent a crisis or a disaster. For this general problem, we cas take as a set of alternatives, a set of different prevention measure and as criteria the various expected improvements by the prevention measures.

Hence with MCDA methods, we may obtain the "best" solution or a ranking of all the prevention measures. A sorting approach can be used also if we want to distinguish efficient measures and nonefficient measures.

Prevention measures to be taken in this problem are often combinations of elementary measures, which involves to consider as the set of alternatives, a set of a portfolio of alternatives. This new specification of the problem and its resolution is an interesting problem in MCDA. Therefore, the results of this research will be a real bridge between risk analysis and MCDA.

#### REFERENCES

- Bana e Costa, C. A., J.-M. D. Corte, and J.-C. Vansnick (2005). On the mathematical foundations of MACBETH. In J. Figueira, S. Greco, and M. Ehrgott (Eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, pp. 409–437. Springer.
- Bisdorff, R. and M. Zam (2009, September 17-18). D4: rationale, concept and implementation of a distributed mcda application designer. *5thDecision Deck Workshop, Telecom Bretagne (France)*.
- Bisdorff, R. and M. Zam (2010, October 6). Agile mcda modelling: D4 meets xmcda. 7thDecision Deck Workshop, Université Paris-Dauphine (France).

- Brans, J.-P., B. Mareschal, and P. Vincke (1984). Promethee: a new family of outranking methods in multicriteria analysis. In J.-P. Brans (Ed.), *Operational Research'84*. North Holland.
- Cailloux, O. and V. Mousseau (2011, 18-22 September). Parametrize a territorial risk evaluation scale using multiple experts knowledge through risk assessment examples. In *proceedings of the annual ESREL Conference*, Troyes, France, forthcoming.
- Chateauneuf, A., M. Grabisch, and A. Rico (2008). Modeling attitudes towards uncertainty through the use of Sugeno integral. J. of Mathematical Economics 44, 1084–1099.
- Dyer, J. (2005). MAUT multiattribute utility theory. In J. Figueira, S. Greco, and M. Ehrgott (Eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, pp. 265–285. Boston, Dordrecht, London: Springer Verlag.
- Figueira, J., S. Greco, and M. Ehrgott (Eds.) (2005). *Multiple Criteria Decision Analysis: State of the Art Surveys*. Kluwer Acad. Publ.
- Figueira, J., V. Mousseau, and B. Roy (2005). ELECTRE methods. In J. Figueira, S. Greco, and M. Ehrgott (Eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, pp. 133–162. Springer.
- Figueira, J. R., S. Greco, and R. Slowinski (2009). Building a set of additive value functions representing a reference preorder and intensities of preference : Grip method. *European Journal of Operational Research 195*(2), 460– 486.
- Greco, S., V. Mousseau, and R. Slowinski (2008). Ordinal regression revisited: Multiple criteria ranking using a set of additive value functions. *European Journal of Operational Research* 51(2), 416–436.
- Keeney, R. L. and H. Raiffa (1976). *Decision with Multiple Objectives: Preferences and Value Tradeoffs*. New York: John Wiley and Sons.
- Marchant, T. (2003). Towards a theory of MCDM: Stepping away from social choice theory. *Mathematical Social Sciences* 45(3), 343–363.
- Mayag, B., M. Grabisch, and C. Labreuche (2009a). A characterization of the 2-additive Choquet integral through cardinal information. In *Proceedings of EURO-FUSE2009 Workshop Preference Modelling and Decision Analysis (CD)*, Pamplona, Spain, pp. 155–160.
- Mayag, B., M. Grabisch, and C. Labreuche (2009b, 20-24 July). A link between the 2-additive Choquet integral and belief functions. In *Proceedings of Int. Conf. of the European Soc. for Fuzzy Logic and Technology (EUSFLAT)*, Lisbon, Portugal, pp. 363–368.
- Meyer, P. (2007). *Progressive Methods in Multiple Criteria Decision Analysis.* Ph. D. thesis, University of Luxembourg and Faculty Polytechnic of Mons.
- Meyer, P. and M. Roubens (2005). Choice, ranking and sorting in fuzzy multiple criteria decision aid. In J. Figueira, S. Greco, and M. Ehrgott (Eds.), *Multiple Criteria Decision Analysis*, pp. 471–506. Kluwer Academic Publishers.
- Mousseau, V., L. Dias, and J. R. Figueira (2006). Dealing with inconsistent judgments in multiple criteria sorting models. *4OR 4*.
- Mousseau, V., J. R. Figueira, L. Dias, C. G. D. Silva, and J. Climaco (2003). Resolving inconsistencies among constraints on the parameters of an MCDA model. *European Journal of Operational Research 147*.

- PPRT (2011). Le plan de prévention des risques technologiques (pprt), guide méthodologique. *Ministère de l'écologie, du développement et de l'aménagement durables*. Available at http://www.developpementdurable.gouv.fr/Maitrise-de-l-urbanisation-PPRT,12775.html.
- Roy, B. (1996). *Multicriteria Methodology for Decision Aiding*. Dordrecht: Kluwer Academic.
- Siskos, Y., E. Grigoroudis, and N. F. Matsatsinis (2005). Uta methods. In J. Figueira, S. Greco, and M. Ehrgott (Eds.), *Multiple Criteria Decision Analysis: State of the Art Surveys*, pp. 297–343. Springer.