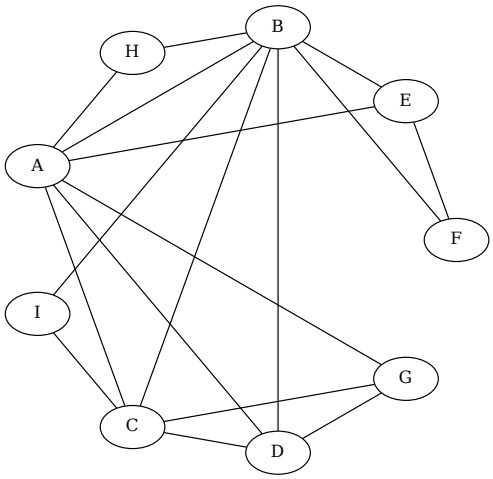
TD 1: Introduction

1 Adjacency Matrices and Lists

Give the adjacency matrix representation and the adjacency list representation of the graph below. Calculate the degree of each vertex.



2 O-notation reminder

Sort the following functions of n in a table where, whenever $f(n) = \Theta(g(n))$ you place f, g on the same row, and whenever $f(n) = o(g(n))$ you place f below g.

$$n^2 + 15n, \frac{n^3}{2}, \log^5 n, (\log n)^{\log n}, \log(n!), 3n \log n, 2^{\sqrt{\log n}}, n^{\log n}, \binom{n}{2}$$

3 Graph Square

If $G = (V, E)$ we define as the square of G, denoted G^2 the graph which has the same vertex set as G and in which two vertices u, v are adjacent if and only if they are at distance at most 2 in G. (This means that u, v are adjacent in G^2 if they are adjacent in G or they have a common neighbor in G.)

Give an algorithm that takes as input G (in matrix or list representation) and outputs G^2 (in the same representation). What is the time complexity of your algorithm?

4 Universal Sink

In a directed graph, a **sink** is a vertex of outdegree 0 (and a **source** is a vertex of indegree 0). A **universal sink** is a sink of indegree $n - 1$. Give an algorithm that takes as input the adjacency matrix of a digraph and outputs a universal sink, or correctly reports that no such vertex exists.

5 Triangle Detection

Give an algorithm which takes as input a graph G (in adjacency matrix or list form) and decides if G contains a triangle, that is, three vertices x, y, z which are pairwise adjacent.

6 Ramsey

Prove that in any group of 6 people, there are either 3 people who all know each other or 3 people who do not know each other. Show that this is false for groups of 5 people.

Generalization: prove that for all k, in any group of 4^k people, there are either at least k who all know each other, or at least k who do not know each other.