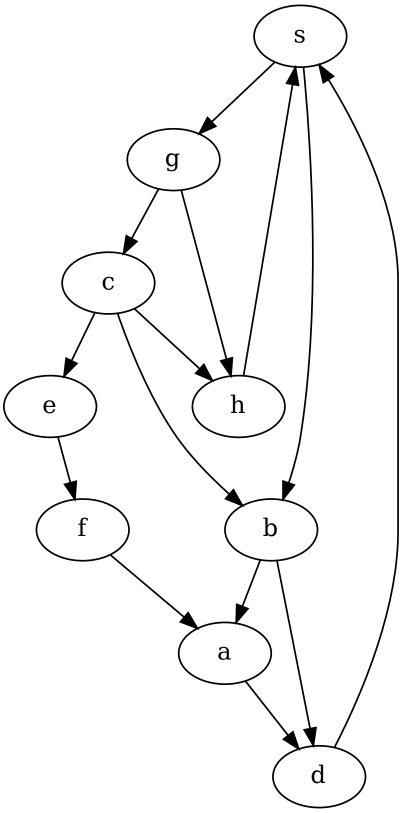
TD 2: BFS

1 Execute BFS

Execute the BFS algorithm on the directed graph below, starting from vertex s. You can assume that adjacency lists are ordered alphabetically. Show the contents of the queue at every iteration, the distances calculated, and the edges of the BFS tree.



2 BFS layers

Show that if we execute BFS on a graph $G = (V, E)$ starting from s, for all $uv \in E$ we have $|d_{BFS}(u) - d_{BFS}(v)| \leq 1$. (Recall that $d_{BFS}(v)$ is the distance computed by BFS for v and we have shown that $d_{BFS}(v) = \mathrm{dist}(s, v)$. Furthermore, we have shown that any two vertices which are simultaneously in the BFS queue have d_{BFS} values which differ by at most 1.).

3 Destroying connectivity

Suppose that in an n-vertex connected undirected graph G, two (given) vertices s, t are at distance strictly greater than $n/2$.

- Prove that there exists a vertex x, such that if we delete x from the graph, then we destroy all paths from s to t.
- Give an algorithm that finds x in time $O(m+n)$ (assuming G is given in the form of adjacency lists).

4 Different BFS trees

For simplicity, we usually assume that adjacency lists are alphabetically ordered. However, using lists in a different order may affect the tree output by the BFS algorithm.

1. Give an example of a graph and two orderings of the vertices such that executing BFS with each ordering produces different trees. Does the ordering of the vertices affect the d_{BFS} values computed?
2. Give an example of a graph $G = (V, E)$, a vertex $s \in V$, and an edge $e \in E$, such that no matter how we order the vertices in the adjacency lists, e will never be part of the tree output by BFS.
3. Give an example of a graph $G = (V, E)$, a vertex $s \in V$, and a set of edges $E_\pi \subseteq E$, such that (i) E_π is a shortest-path tree from s in G (ii) no possible ordering can make BFS output the tree E_π.

5 Find a cycle through an edge

Give an algorithm which takes as input a graph $G = (V, E)$, and an edge $uv \in E$ and decides in linear time whether there exists a cycle in G that traverses the edge uv.