Exact Algorithms

Homework 4

Lecturer: Eunjung KIM

1. Consider a parameterized problem CLUSTER VERTEX DELETION defined as follows.

CLUSTER VERTEX DELETION (CVD)

Input: a graph $G = (V, E)$, a nonnegative integer k.

Parameter: k.

Task: find a vertex subset $X \subseteq V(G)$ of size at most k such that $G - X$ is a cluster. Here, a cluster graph is a disjoint union of complete graphs (=cliques).

COMPRESSION CVD

Input: a graph $G = (V, E)$, a vertex set $X \subseteq V$ such that $G - X$ is a cluster graph.

Parameter: k.

Task: find a vertex subset $Y \subseteq V$ such that $G - Y$ is a cluster graph and $|Y| < |X|$.

1-(a). Present a branching-based FPT-algorithm for CVD running in time $O^*(3^k)$.
Hint: Fill in the blank to make the following statement holds. "A graph G is a cluster graph if and only if G does not contain (   ) as an induced subgraph.

1-(b). Design an algorithm for COMPRESSION CVD running in time $O^*(2^{|X|})$. Don't need to show correctness of the algorithm.
Hint: use a polynomial-time algorithm for weighted matching problem as a blackbox.

2. We say that a graph class (=a set of graphs) $\mathcal{F}$ is *hereditary* if for every $G \in \mathcal{F}$, any induced subgraph of G is also in $\mathcal{F}$.

2-(a). Show that the graph class of all edgeless graphs is hereditary. Show the same statement for the class of all acyclic graphs, and all cluster graphs.

2-(b). Use 2-(a) to prove the following statement: given an algorithm $\mathcal{A}$ for the COMPRESSION CVD, one can solve CVD in time $\mathcal{O}^*(2^k)$ by performing $\mathcal{A}$ at most n times. You can use the algorithm of 1-(b).

★3. Given a graph G and an integer k, EDGE BIPARTIZATION asks to find an edge subset $D \subseteq E(G)$ of size at most k so that $G - D$ becomes a bipartite graph. Design an iterative compression algorithm running in time $O^*(2^k)$.

♣ Submit your solution via email (eunjungkim78@gmail.com) by 15 Feb 2019, midnight.
♣ Questions with ★ can be worked together with a colleague. But please write the solution by yourself.