1. Consider a parameterized problem \textsc{Cluster Vertex Deletion} defined as follows.

\textsc{Cluster Vertex Deletion (CVD)}

\textbf{Input:} a graph \( G = (V, E) \), a nonnegative integer \( k \).

\textbf{Parameter:} \( k \).

\textbf{Task:} find a vertex subset \( X \subseteq V(G) \) of size at most \( k \) such that \( G - X \) is a cluster. Here, a cluster graph is a disjoint union of complete graphs (=cliques).

\textsc{Compression CVD}

\textbf{Input:} a graph \( G = (V, E) \), a vertex set \( X \subseteq V \) such that \( G - X \) is a cluster graph.

\textbf{Parameter:} \( k \).

\textbf{Task:} find a vertex subset \( Y \subseteq V \) such that \( G - Y \) is a cluster graph and \(|Y| < |X|\).

1-(a). Present a branching-based FPT-algorithm for CVD running in time \( O^*(3^k) \).

Hint: Fill in the blank to make the following statement holds. "A graph \( G \) is a cluster graph if and only if \( G \) does not contain ( ) as an induced subgraph.

1-(b). Design an algorithm for \textsc{Compression CVD} running in time \( O^*(2^{|X|}) \). Don’t need to show correctness of the algorithm.

Hint: use a polynomial-time algorithm for weighted matching problem as a blackbox.

2. We say that a graph class (=a set of graphs) \( \mathcal{F} \) is hereditary if for every \( G \in \mathcal{F} \), any induced subgraph of \( G \) is also in \( \mathcal{F} \).

2-(a). Show that the graph class of all edgeless graphs is hereditary. Show the same statement for the class of all acyclic graphs, and all cluster graphs.

2-(b). Use 2-(a) to prove the following statement: given an algorithm \( A \) for the \textsc{Compression CVD}, one can solve CVD in time \( O^*(2^k) \) by performing \( A \) at most \( n \) times. You can use the algorithm of 1-(b).

\textstar\textbullet 3. Given a graph \( G \) and an integer \( k \), \textsc{Edge Bipartization} asks to find an edge subset \( D \subseteq E(G) \) of size at most \( k \) so that \( G - D \) becomes a bipartite graph. Design an iterative compression algorithm running in time \( O^*(2^k) \).

\textstar Submit your solution via email (eunjungkim78@gmail.com) by 15 Feb 2019, midnight.

\textstar Questions with \textstar can be worked together with a colleague. But please write the solution by yourself.