2°

POLITECNICO DI MILANO

Methods and Models for Decision Making

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God in 7 steps:

- A decision problem involve a choice
- Usually you have a "real" decision problem (and not an "ideal" one)
  - actors (conflicts)
  - criteria (trade-off)
  - lack of information
- There are tools for decision aiding

  ↳ abstraction / analysis / synthesis

Index:

Mental models

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Examples (Theler, 1991)

1

Colorni won
- 200 € (at a homely bingo)
- 800 € (at a parish bingo)

Luè won 1.000 € (at a Politecnico bingo)

who is more satisfied ?

2

Colorni has to pay
- 200 € for ICI (a house owner tax)
- 800 € for IVA (VAT tax)

Luè has to pay 1.000 € for IRPEF (another tax)

who is less unhappy ?

3

Colorni has an accident and pays 800 €
but he has a reimbursement from the insurance 200 €

Luè's car has been striped, he has to pay 600 €

who is less unhappy ?

4

Colorni receives a production bonus for 1.000 €
but he discovers a 300 € debt

Luè receives a production bonus for 700 €

who is more satisfied ?

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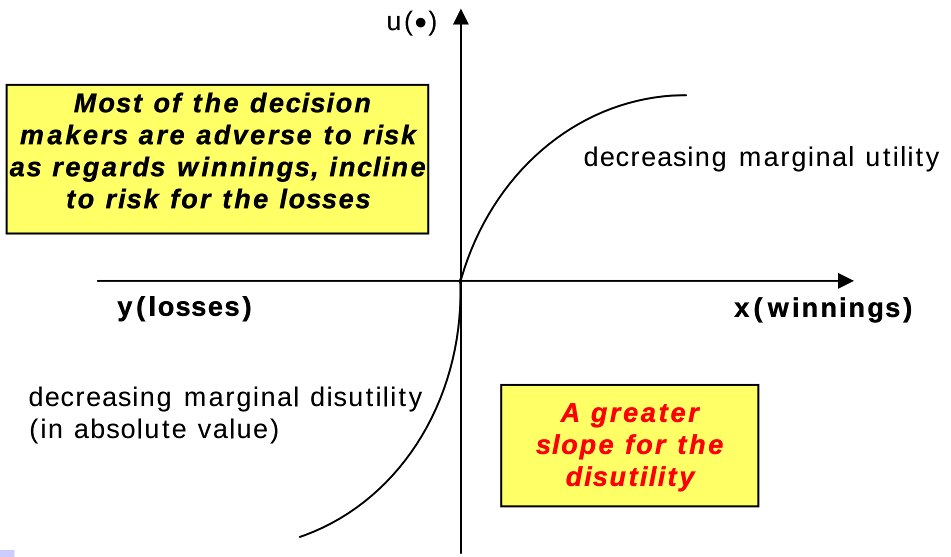
Mental models – 1

1. There is a dissociation for the **winnings**
2. There is an aggregation for the losses
3. There is a dissociation between **low winnings** and **high losses**
4. There is a aggregation between **high winnings** and **low losses**

Dissociation = overestimation
Aggregation = underestimation

A **subjective** utility function (Bernoulli, S. Pietroburgo, 1738)



$u(x_1 + x_2) < u(x_1) + u(x_2)$ → **UTILITY**

$u(y_1 + y_2) > u(y_1) + u(y_2)$ → **DISUTILITY**

(warning ! values < 0)

Risk and perception – 1

Protocol A	20% immediate death 80% increase expected lifetime by 30 years	
Protocol B	100% increase expected lifetime by 18 years	

Better A or B ?

Anticancer therapy
on "group X" at European
Institute of Oncology (IEO)

Patients of about 40 years, **with expected life of 3-6 months**

Protocol C	80% immediate death 20% increase expected lifetime by 30 years	
Protocol D	75% immediate death 25% increase expected lifetime by 18 years	

Better C or D ?

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Risk and perception – 2

- **Group X** → Only 1 patient in 4 reacts positively, then who react can choose between E and F

Protocol E 20% immediate death
80% increase expected lifetime by 30 years

Protocol F 100% increase expected lifetime by 18 years

Anticancer therapy on "group X" at IEO

- Comment 1 → this situation (E-F) is the same of the previus one (C-D)
- Comment 2 → The decision-makers "clears" the information context (only 1 in 4 ...) and decides **between the proposed options**

The mental model may depend on the communication

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Definitions – 1

(a) Objective probability (frequentist) → **p = f / t**

→ ratio of the number of favorable cases (f) to number of total cases (t)

→ applicable only to problems with repeated events ∞ (many) times

(b) Subjective probability → **p = … (?)**

→ ***personal*** assessment of the ratio of favorable cases to total cases

→ everyone can assess its ***own*** probability to every casual event, this represents his degree of confidence

→ how can you measure this probability ?

by means of a lottery

- **Examples**

Random events: what probability ?

O = objective probability
S = subjective probability

1. Probability of having two pairs and changing one card … O S
2. Probability that my number wins to the lottery "Lotteria Italia" O S
3. Probability of rain tomorrow in Milan O S
4. Probability that (having 60 years and being in good health) I will be alive in 20 years O S
5. Probability that, from a survey of 2000 people done before the elections, I guess the party who will govern Italy O S
6. Probability that if the avian influence hits Italy, the vaccine is effective O S
7. Probability that Soldatino wins the Gran Premio degli Assi a Tordivalle (Febbre da cavallo, a movie of 1982) O S
8. Other examples proposed by you …

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Axioms of probability theory

A1 - Probability **p(e)** of an event (e): value between
- 0 (impossible)
- 1 (certain)

A2 - Complementary probability (the event does not occur): **1-p(e)**

A3 – For events (e_1, e_2, ..., e_k) that are mutually exclusive : $p(e_1 \text{ OR } \ldots \text{ OR } e_k) = p(e_1) + \ldots + p(e_k)$

A4 – For 2 independent events (e_1, e_2) : $p(e_1 \text{ AND } e_2) = p(e_1) * p(e_2)$

A5 – For 2 non-indipendent events (e_1, e_2):

$$p(e1 / e2) = \frac{p(e1 \text{ AND } e2)}{p(e_2)} = \frac{p(e_2 / e_1) * p(e_1)}{p(e_2)}$$

→ conditional probability (Bayes, 1763)

A6 – If an event has an expected value v_0 then a sequence of **n** repetitions has an expected value of $n * v_0$

(see Lottery L1)

An example follows

The barometer (an example)

w = state of nature

w1 = good weather

w2 = bad weather

p(w)

w1	w2
.80	.20

p(y)

y1	.55
y2	.25
y3	.20

p(w,y)

	w1	w2
y1	.50	.05
y2	.20	.05
y3	.10	.10

p(w/y)

	w1	w2
y1	.91	.09
y2	.80	.20
y3	.50	.50

y1 = clear

y2 = variable

y3 = rain

p(y/w)

	w1	w2
y1	.63	.25
y2	.25	.25
y3	.12	.50

p(y/w) → in this case does not make much sense

Definitions – 2

Lottery

Given a ***certain event R1*** of which the decision-maker knows how to estimate the utility (that is, his level of satisfaction), if exists an ***uncertain event R2*** with a higher utility, the decision-maker is able to compare the utility of R1 (obtainable with certainty) with the ***equivalent utility*** of obtaining R2 with probability p and obtaining nothing (the null event) with probability (1-p). ***Determining p*** is a prerogative of the decision-maker.

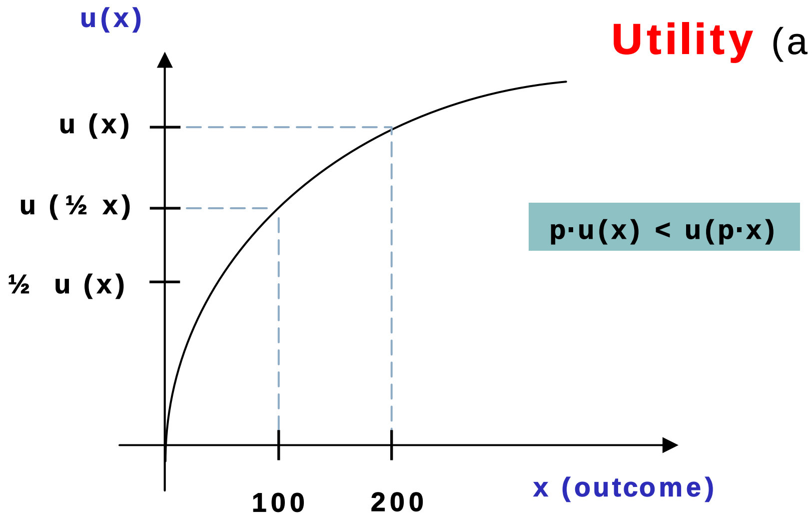
Utility (see following slide)

State of nature

The set of variables that are ***not controlled*** by the decision-maker, but that influence the final result (also known as "exogenous variables").

Expected value vs. Expected utility

Utility (a subjective level of satisf.)



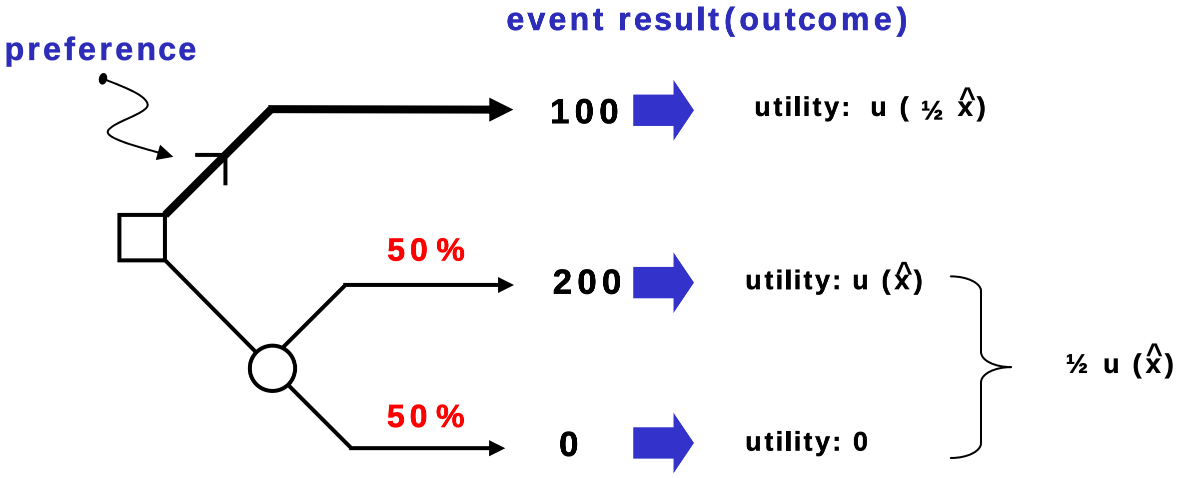
RISK AVERSION: the decision-maker prefers certain 100 € than the lottery with 200 € at 50% and 0 € to 50%

CERTANTY EQUIVALENT ē : value that makes the sure event ē equivalent to the lottery with x€ at 50% and 0€ at 50%

(i) ē = 30

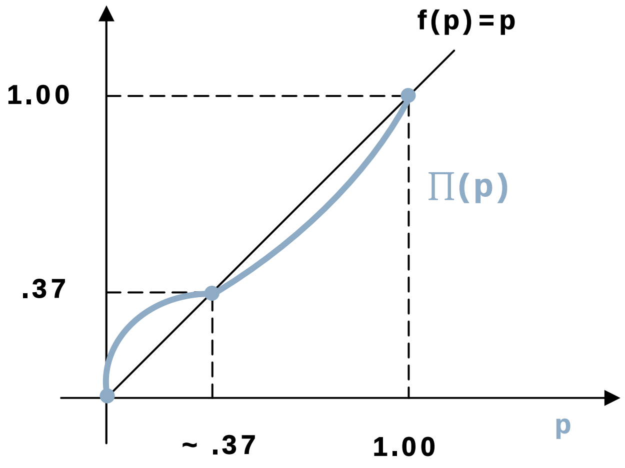
(ii) ē = 90

who is more risk-adverse ?



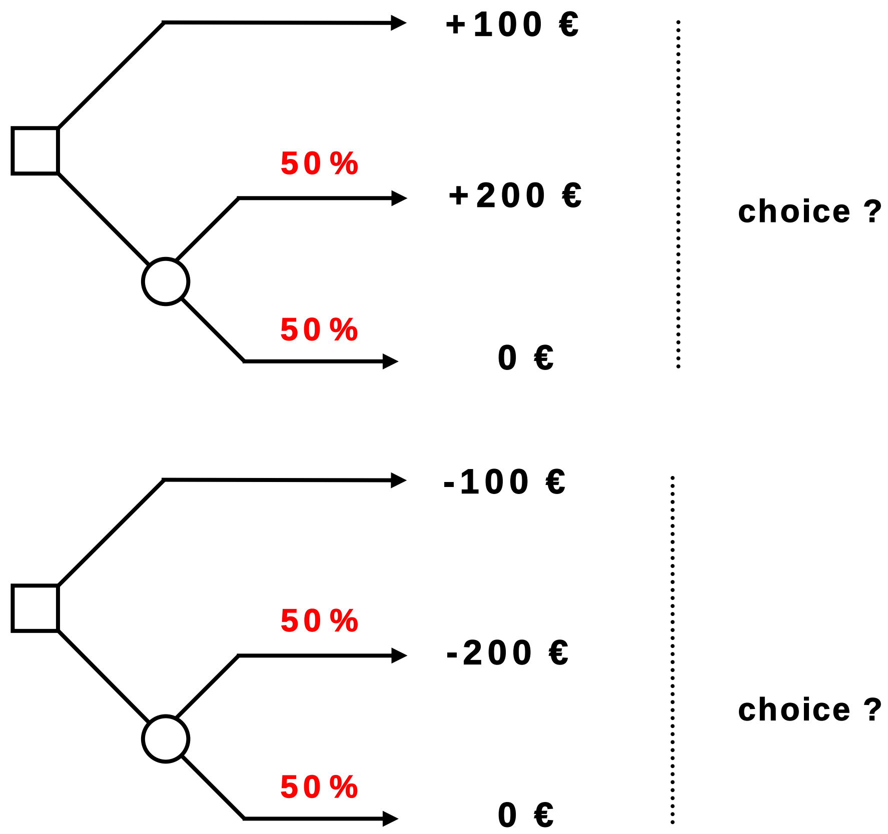
Mental models – 2

[Tversky & Kahneman]



Representation of the "mental" weight ∏(p) assigned to different probabilities p

- over-estimation of low p values
- under-estimation of high p values



The majority of the decision makers is:

- risk-adverse in case of winnings
- risk-inclined in case of losses

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Frame effect

- Avian influence (possible death)
- Group at risk: 600 people

Protocol A 200 people will survive

Protocol B
- with p=1/3 600 will survive
- with p= 2/3 nobody will survive

Better A or B ?

Protocol A 400 people will die

Protocol B
- with p=1/3 nobody will die
- with p= 2/3 600 will die

Better A or B ?

- **Aversion to the risk in case of winnings**
- **Propensity for risk in case of losses**

Choice vs. rejection

Shafir (1993)

Cause for divorce, with the choice for the custody of the only child →

Parent A	Parent B
Average Income	High income
Normal health	Small health problems
Regular working hours	Many business trips
Acceptable relationship with the child	Very close relationship with the child
Stable social life	Extremely active social life

Group 1

→ Which parent would you give child's custody?

Group 2

→ Which parent would you reject the child's custody to?

Info on the parent B are strongly polarized

Example (more)

- ***Preference*** for an alternative vs. ***Rejection*** of an alternative

  ONE **BETWEEN** MANY — ONE **AGAINST** MANY

- If there are 2 alternatives — the two situation should coincide (but it is not always true)

- ***Choiche*** vs ***Non-choice***

  → causes for not chosing
  - lack of information (however → ***experiments***)
  - difficulty in appreciating the differences → ***incomparability*** → Outranking methods (Electre)

- Often the difficulty of settling the conflict is overcomed → introducing other alternatives (to facilitate the comparison)

Conclusions

- Bibliography:

1. M.Piattelli Palmarini, "Psicologia ed economia delle scelte" (in Italian), Codice, 2005.

2. D. Kahneman, A. Tversky, "Choices, Values, and Frames", Cambridge Un. Press, 2000

- Test

T T T T T C C C C C — which sequence is the most probable one?

C T T C T T T C T T — which final result (T/C) is the most probable one?

C: cross   T: head

- Two problems

Two problems

1. Example of Bayes

- a woman at a doctor → nodule
- examination → possibile tumor (10 %)
- mammography < reliable at 90 % / wrong answer at 20 %
- "positive" result

what is the probability to have a tumor ?

y1 = positive result
y2 = negative result

w1 = tumor
w2 = healthy

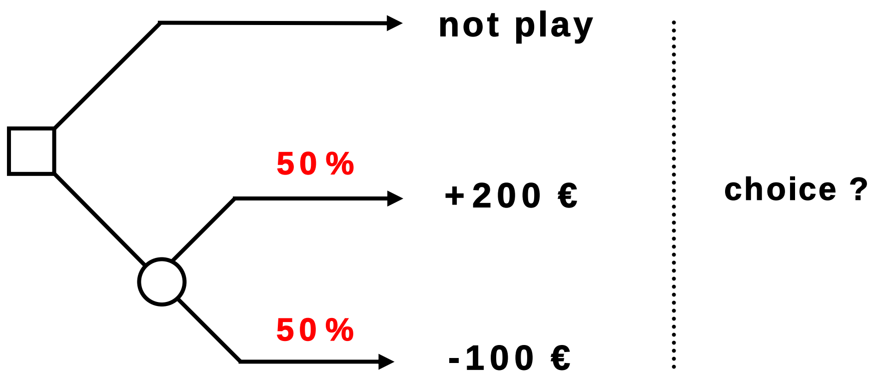
	w1	w2
y1	.09	.18
y2	.01	.72
p(w)≡	.10	.90

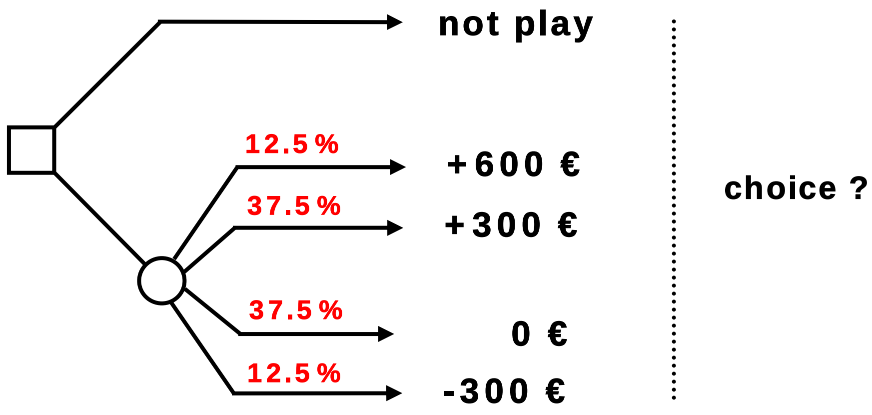
.27
.73

.33	.67
.02	.98

probability = 1/3

2. Example by Tversky (1992)





D & D
(Design & Decision)

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Decision Aid (DA) in design context

- Why Decision Aid (DA) in this context ?

- Design of what ?

    i. **PRODUCT**

    ii. **SERVICE**

    iii. **PROCESS**

    iv. … (other) … ?

Case 1 - Product

- Tha nail holder avoiding to hurt one's hand while hammering

  ↓

  the objective

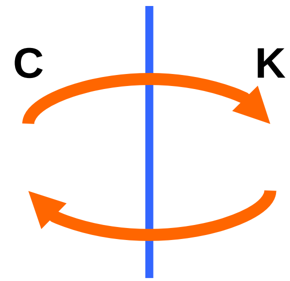
- A great number of alternatives!
  - hand protection
  - fore - hole
  - …

- From a large amount of knowledge to a (limited) set of alternatives

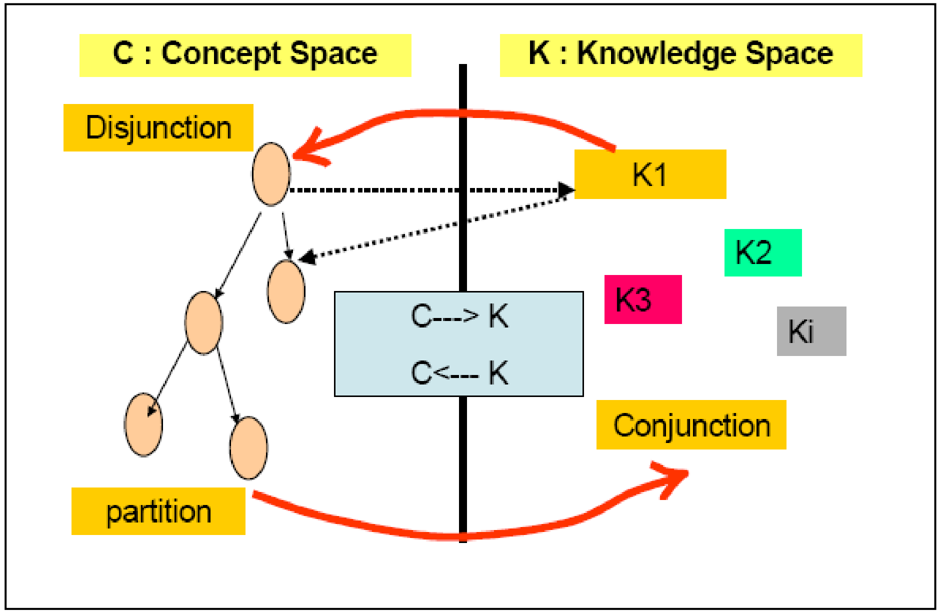
Focus → generating possible solutions

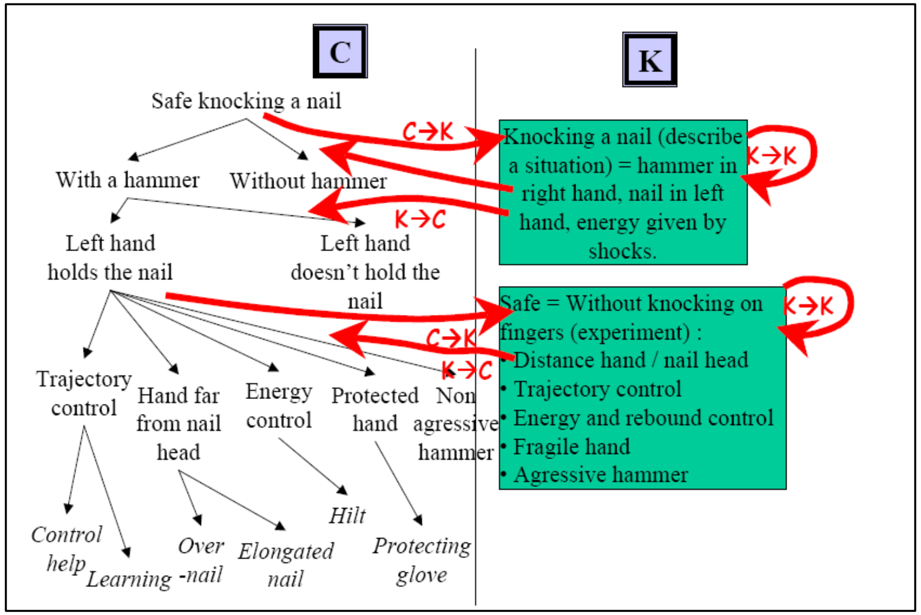
Case 1 – Knowledge vs Concept

- Knowledge ➡ a set (space) of propositions that are true or false
- An object ➡ defined by a set of attributes (and by their possible values)
- Space K ➡ the cartesian product of the attributes
- Space C ➡ a space where to *add/eliminate/change* the attributes
- From space K to space C and vice-versa



Case 1 – The C-K theory

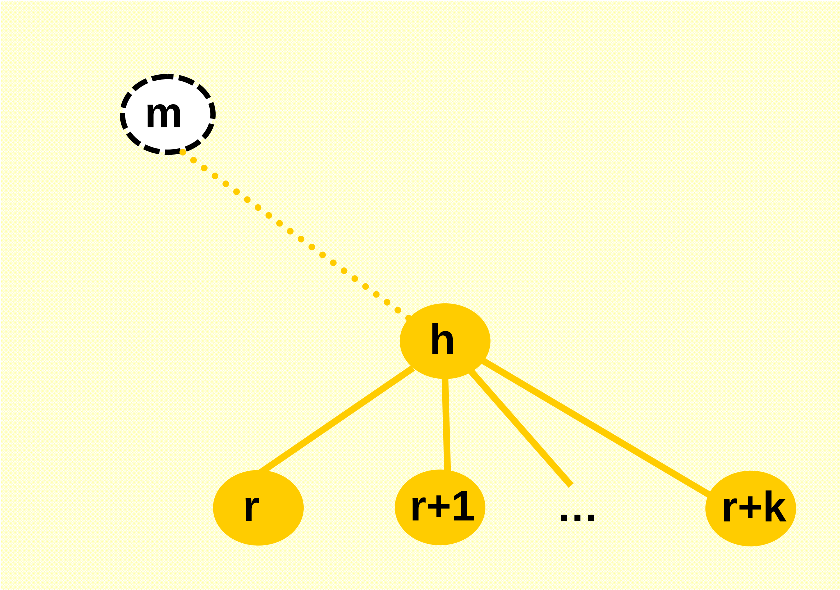




- ➢ The C-K dynamics
- ➢ Nail holder phase 1-a…

Link to …

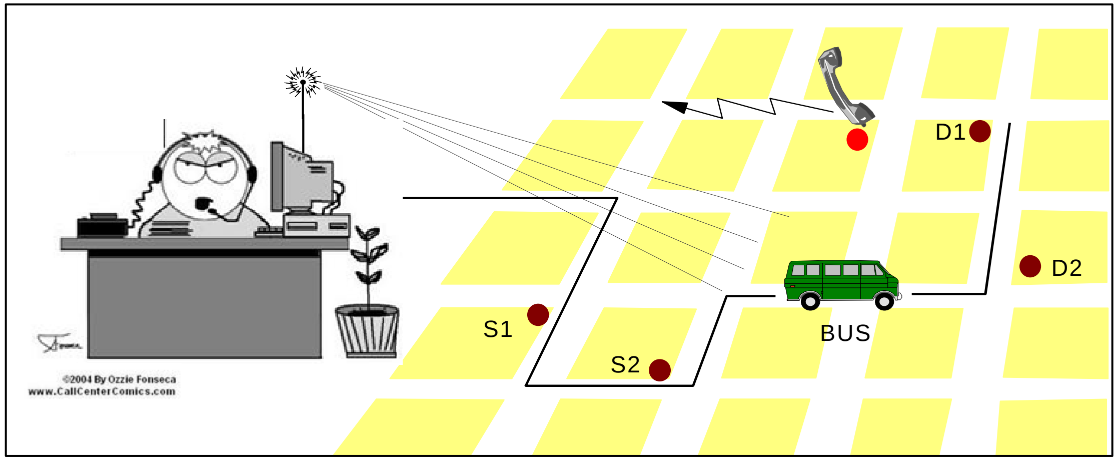
Case 1 – A branch tree



- A node:
  - a predecessor (father)
  - more successors (children)
- In general
  - a condition (constraint) is “inherited” by the father
  - the children describe a partition of the “world” represented in the node $(sol_r \cup sol_{r+1} \cup \ldots \cup sol_{r+k} \equiv sol_h)$
- The role of the bounds
  - B&B (branch and bound) methods

Case 2 - Service

- A (public) service for weak demand (irregular) mobility
  - a condition
- A “dial-a-ride” system



- What is a good service?
  - low cost ?
  - high coverage ?
  - quick door-to-door ?
  - …

Focus → different point of view

http://projectapps.vtt.fi/Connect/portal/alias__Rainbow/lang__en/tabID__3342/DesktopDefault.aspx

Case 2 – A specific ITS (intelligent transport system)

- **Dial-a-ride (DaR)** service
  - a trade-off between
    - public regular service
      - low cost
      - low flexibility
    - personal car (taxi)
      - hight cost
      - hight flexibility

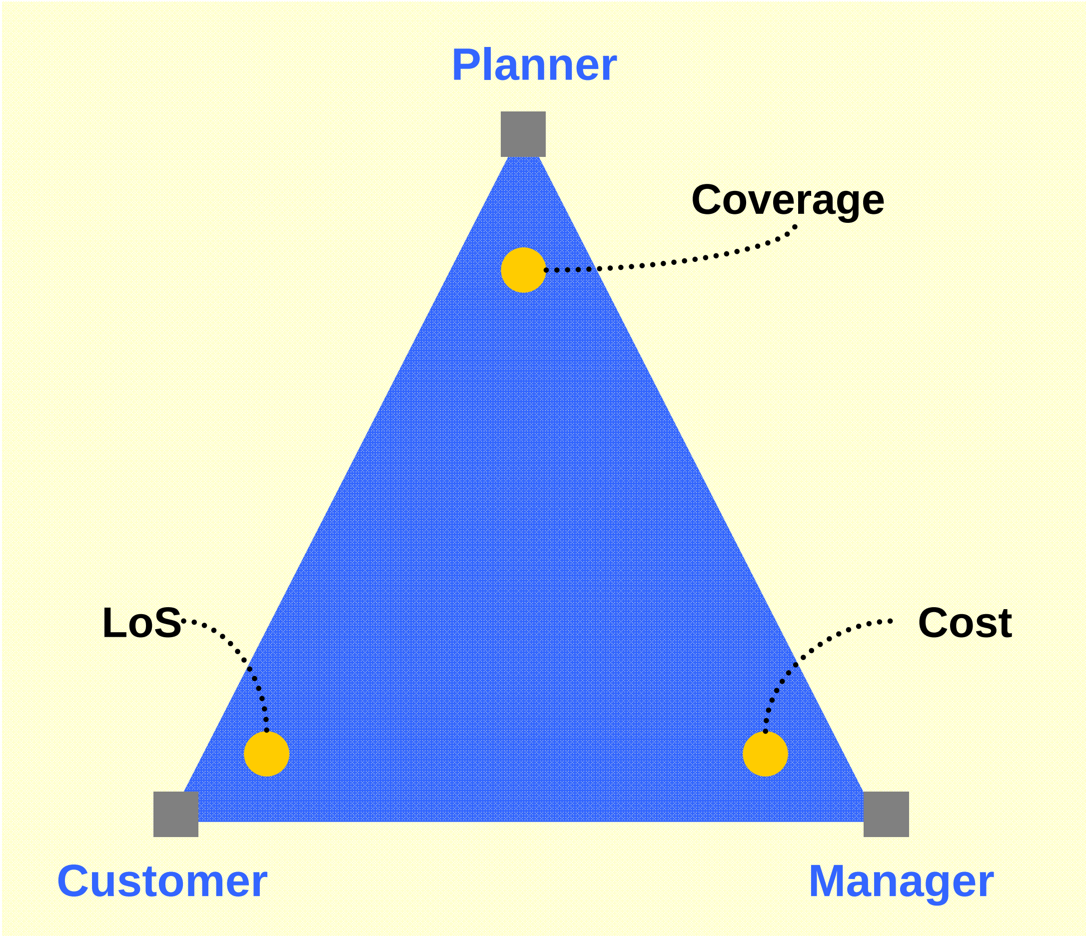
- A classical balance between economics and quality

- DaR service:
  - when-where the demand appears
  - useful in situation of
    - weak demand
    - night hours
    - particular customers
  - safe (door-to-door)

http://www.tempi.piacenza.it/prontobus/prontobus.asp#come_nasce_prontobus

Case 2 – The trade-off approach

- Three points of view:
  i. the planner objective is the area coverage
  ii. the manager objective is the cost control
  iii. the customer objective is the Level of Service (LoS)



- What measures ?

- How compare them ?

Case 2 – The conceptual path

1. Definition of (multiple) objectives
2. Choice of the set of indicators (each with its measure unit)
3. Matrix of effects/impacts (quantitative and qualitative)
4. From indicators to utilities

   ➘ the value functions (…)

5. Matrix of evaluation (values in a common scale)
6. … (see in the following)

Case 3 - Process

- The urban plan of a (small) town
- A set of coordinate actions ➡ the need
- Identification of elementary actions
- Evaluation of the effects (costs, impacts, …)

Focus → analysis of the combinations of (elementary) actions

Case 3 – The effect of a combination

- Example 1 ⟶ the value of the ***refugee suitcase***
  - a set of items
  - each item has a value and a weight
  - the refugee can choose a subset of them
  - there is a constraint of total weight supported
  - how does the refugee choose ?

- Example 2 ⟶ the value of the ***beautycase***
  - toothbrush (value v_b)
  - toothpaste (value v_p)
  - other objects (not important…)
  - the value V of the beautycase is the sum or … ?

Case 3 – Accumulation (the sum operator)

- Model:

item	Vi	Wi
1	50	10
2	80	8
3	20	5
4	60	5

max f = $50x_1 + 80x_2 + 20x_3 + 60x_4$

$10x_1 + 8x_2 + 5x_3$ x3 $+ 5x_4 \leq W$

$x_i = 0,1$

W (total weight supported) = 16

ADDITIVE MODEL

- Decision aid: an algorithm
  - exact : 2^4 combinations (see *)
  - heuristic (ranking by …)

item₄ → 60/5 = 12
item₂ → 80/8 = 10
item₁ → 50/10 = 5
item₃ → 20/5 = 4

then…

Case 3 – Combinatories

values: 50, 80, 20, 60
weight: 10, 8, 5, 5 (W=16)

#	x_1	x_2	x_3	x_4	f
1	0	0	0	0	0
2	0	0	0	1	60
3	0	0	1	0	20
4	0	0	1	1	80
5	0	1	0	0	80
6	0	1	0	1	**140**
7	0	1	1	0	100
8	0	1	1	1	**n.f.**

#	x_1	x_2	x_3	x_4	f
9	1	0	0	0	50
10	1	0	0	1	110
11	1	0	1	0	70
12	1	0	1	1	**n.f.**
13	1	1	0	0	**n.f.**
14	1	1	0	1	**n.f.**
15	1	1	1	0	**n.f.**
16	1	1	1	1	**n.f.**

(*)

Case 3 – Synergy (some operators)

- A set Ω of elements

- A function f such that
  - $f(\Phi) = 0$
  - $f(A) \leq f(B)$ if $A \subseteq B$

  (the function is monotone non decreasing

- Choquet integral (a rough presentation):
  - $\Omega = \{ x_1, x_2, x_3 \}$
  - $f(\Omega) = \alpha f(x_1) + \beta f(x_2) + \chi f(x_3) + \delta f(x_1, x_2) + \ldots + \sigma f(x_1, x_2, x_3)$

    with $\alpha, \beta, \ldots, \sigma$ weights

link !

- OWA (Ordered Weighted Average):
  - order the elements following their value
  - define different weights with respect to the rank position
  - example 1 → weight 1 for the higher
  - example 2 → weight 0 for the extremes → the gym. jury

wikipedia ...

Case 3 – A regional plan

- Key-point: a plan is a set of coordinated actions

- So ➡
  - feasible actions
  - combinations of actions
  - synergies or cumulus of effects
  - alternatives (feasible)
  - effects (of each alternatives to the set of indicators)
  - …

link !

Decision Aid (DA) in design context

- Why Decision Aid (DA) in this context ?
- Design of what ?

i. **PRODUCT** ➡ from (distributed) knowledge to concept

(generating-analyzing possible sol.)

ii. **SERVICE** ➡ consider the different actors & their points of view

iii. **PROCESS** ➡ combination of (elementary) actions