International Doctoral School Algorithmic Decision Theory: MCDA and MOO

Lecture 5: Metaheuristics and Applications

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Overview

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1. Metaheuristics
2. Finance
   - Portfolio Selection
3. Transportation
   - Train Timetable Information
   - Crew Scheduling
4. Medicine
   - Radiotherapy Treatment Design
5. Telecommunication
   - Routing in IP Networks
6. Conclusion

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Heuristics versus Metaheuristics

Heuristic: Technique which seeks near-optimal solutions at a reasonable computational cost without being able to guarantee optimality ... often problem specific (Reeeves 1995)

Examples: Multiobjective greedy and local search (Ehrgott and Gandibleux 2004, Paquete et al. 2005)

Metaheuristic: Iterative master strategy that guides and modifies the operations of subordinate heuristics by combining intelligently different concepts for exploring and exploiting the search space ... applicable to a large number of problems. (Glover and Laguna 1997, Osman and Laporte 1996)

Examples: MOEA, MOTS, MOSA etc.

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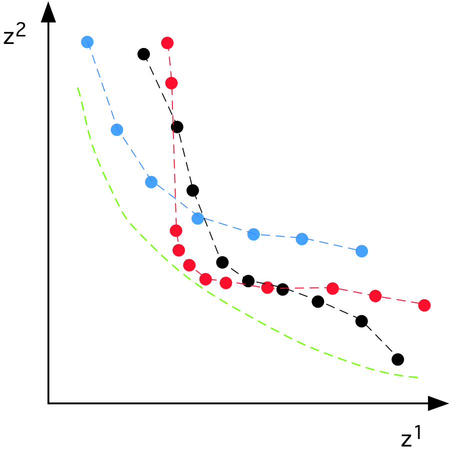

Which approximation is best?

The Quality of Heuristics

- Cardinal and geometric measures (Hansen and Jaszkiewicz 1998)
- Hypervolumes (Zitzler and Thiele 1999)
- Coverage, uniformity, cardinality (Sayin 2000)
- Distance based measures (Viana and de Sousa 2000)
- Integrated convex preference (Kim et al. 2001)
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Timeline for Multiobjective Metaheuristics

- Evolutionary Algorithms
  - 1984: VEGA by Schaffer
- Neural Networks
  - 1990: Malakooti
- Neighbourhood Search Algorithms
  - 1992: Simulated Annealing by Serafini
  - 1992/93: MOSA by Ulungu and Teghem
  - 1996/97: MOTS by Gandibleux et al.
  - 1997: TS by Sun
  - 1998: GRASP by Gandibleux et al.
- Hybrid and Problem Dependent Algorithms
  - 1996: Pareto Simulated Annealing by Czyzak and Jaszkiewicz
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- Main principles
  1. Population of solutions
  2. Self adaptation (independent evolution)
  3. Cooperation (exchange of information)
- Main problems
  1. Uniform convergence (fitness assignment by ranking and selection with elitism)
  2. Uniform distribution (niching, sharing)
- Huge number of publications, including surveys, books, EMO conference series
- Few applications to MOCO problems

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Multiobjective Neighbourhood Search Algorithms

Multiobjective Simulated Annealing

- Initial solution
- Neighbourhood structure
- Scalarizing function $s(z(x), \lambda)$
- Set of weights (directions) λ
- Simulated annealing based on s
- Merge sets of solutions

Multiobjective Tabu Search

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Hybrid Algorithms

1. EA components in NSA (to improve coverage)
   - Pareto Simulated Annealing (Czyzak and Jaszkiewicz 1996)
     Use set of starting solutions
     SA uses information from set of solutions
   - TAMOCO (Hansen 1997, 2000)
     Use set of starting solutions
     TS uses information from set of solutions
2. NSA strategies inside EAs (to improve convergence)
   - MGK (Morita et al. 1998, 2001)
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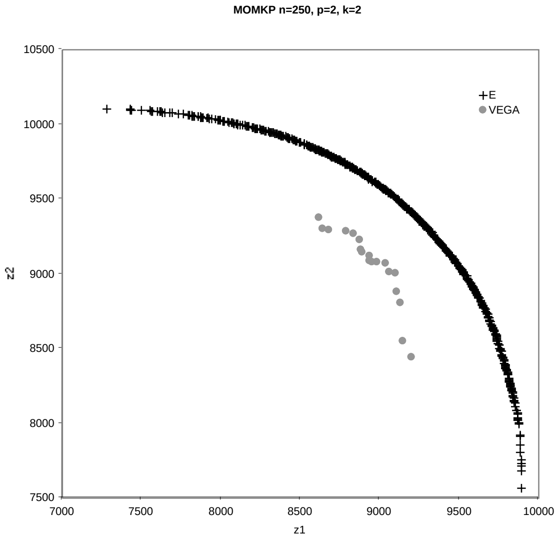
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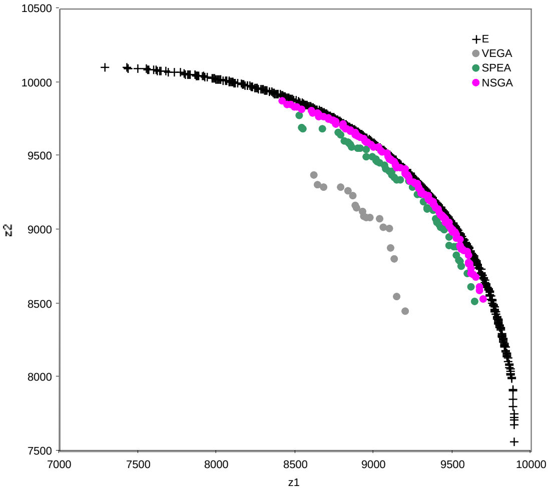

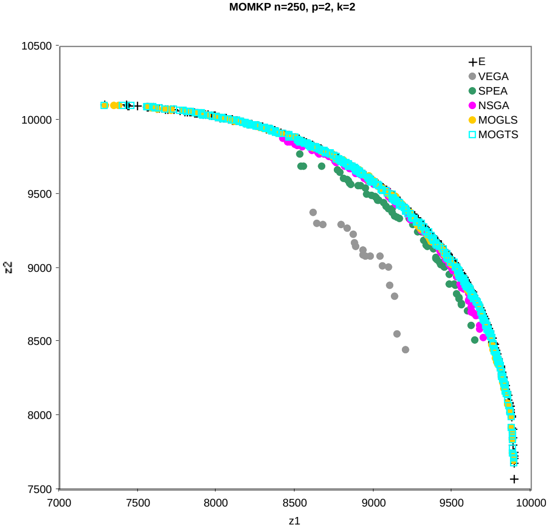
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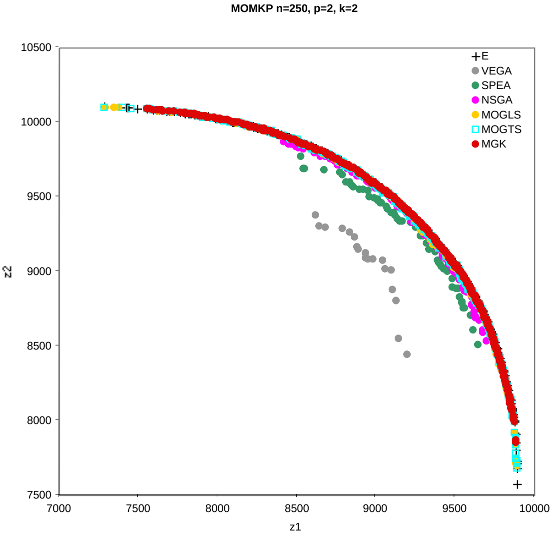
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Hybrid Algorithms

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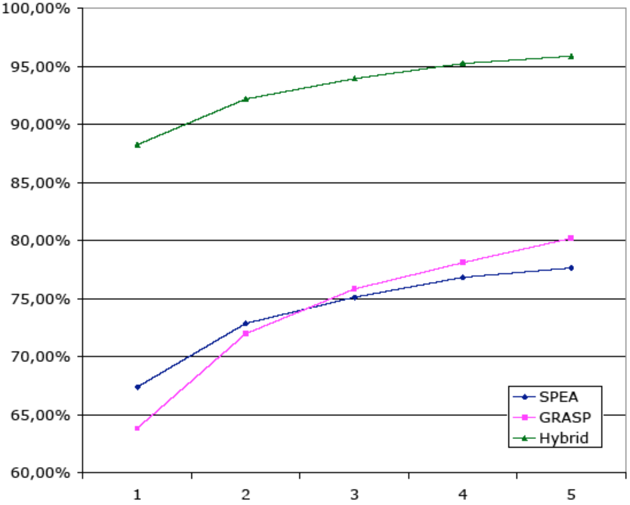
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1
2
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4
5
SPEA
GRASP
Hybrid

Hybrid Algorithms

4. Combination of EA, NSA and problem dependent components

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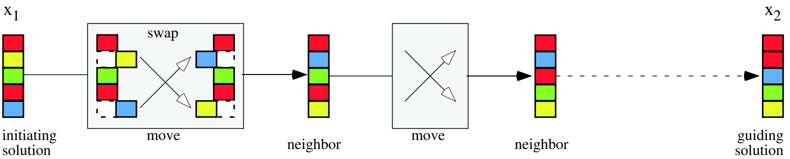
  Use crossover and population as EA component
  Use path-relinking as NSA component
  Use X_{SE_m} and bound set as problem dependent component



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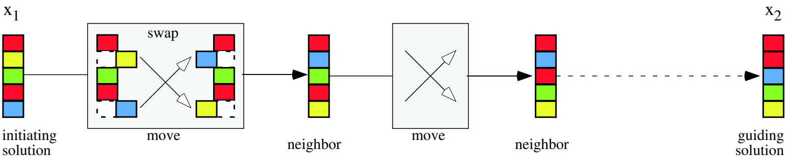

Hybrid Algorithms

5. Combination of heuristics and exact algorithms
   - Gandibleux and Fréville 2000
     Use tabu search as heuristic
     Use cuts to eliminate search areas where (provably) no efficient solutions exist
   - Przybylski et al. 2004
     Use two phase method as exact algorithm
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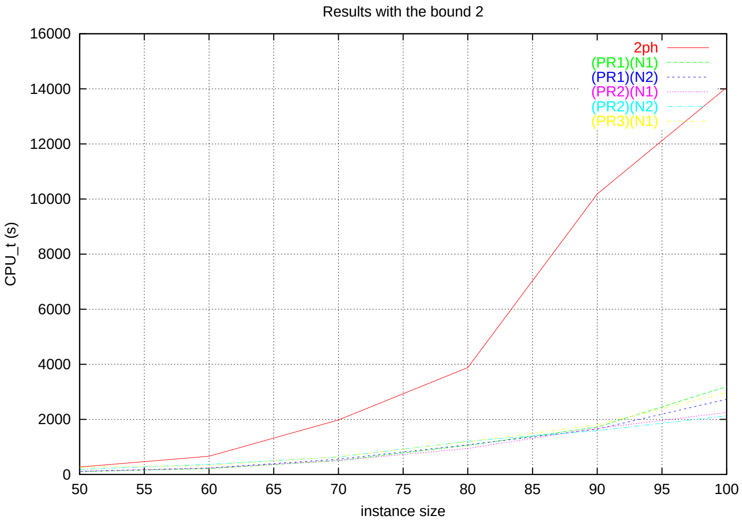
Results with the bound 2
16000
14000
12000
10000
8000
6000
4000
2000
0
CPU_t (s)
50
55
60
65
70
75
80
85
90
95
100
instance size
2ph
(PR1)(N1)
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New Metaheuristic Schemes for MOCO

1. Ant colony optimization (Iredi et al. 2001, Gravel et al. 2002, Doerner et al. since 2001)
2. Scatter search (Beausoleil 2001, Molina 2004, Gomes da Silva et al. 2006, 2007)
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More information: Ehrgott and Gandibleux 2005, 2007

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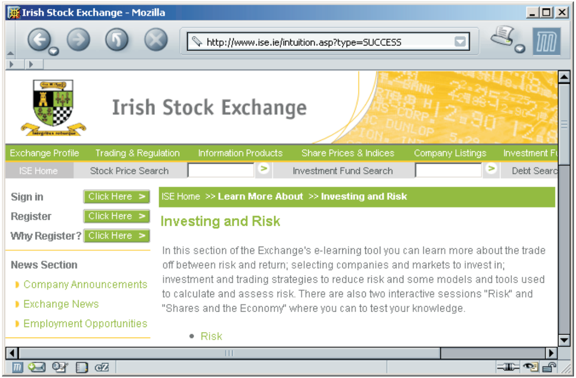
Stock Exchange, Risk, and Return



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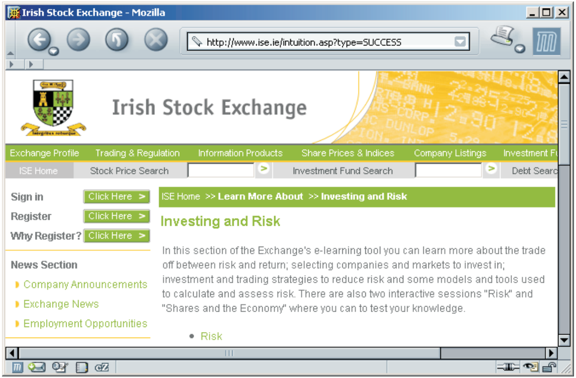

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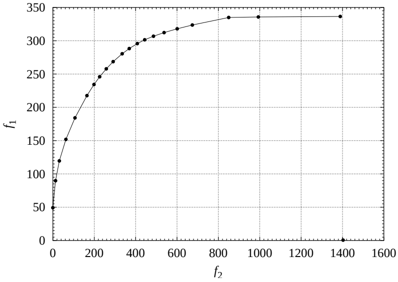
Portfolio Selection

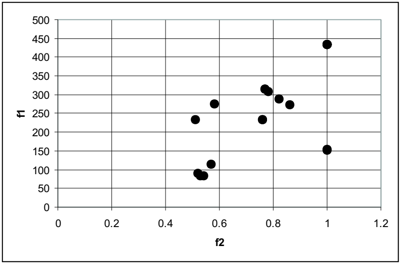
Markowitz 1952

$$
\begin{aligned}
\max f_1(x) &= \mu^T x \\
\min f_2(x) &= x^T \sigma x \\
\text{subject to } e^T x &= 1 \\
x &\geqq 0
\end{aligned}
$$



Standard and Individual Investors

Why do investors not buy efficient portfolios?



Markowitz model assumes "standard" investor

Individual investor has more objectives (Steuer et al. 2006, Ehrgott et al. 2004)

$$\begin{aligned} \max f_1(x) &= \mu_1^T x \\ \min f_2(x) &= x^T \sigma x \\ \max f_3(x) &= \mu_3^T x \\ \max f_4(x) &= d^T x \\ \max f_5(x) &= s^t x \\ \text{subject to } e^T x &= 1 \\ x &\geqq 0 \end{aligned}$$

Standard and Individual Investors

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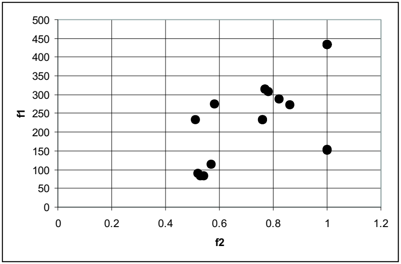

Markowitz model assumes "standard" investor

Individual investor has more objectives (Steuer et al. 2006, Ehrgott et al. 2004)

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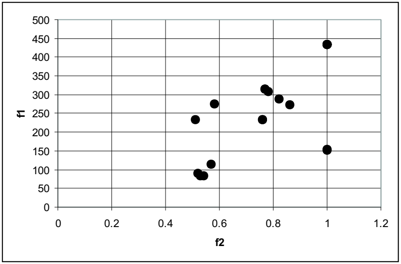

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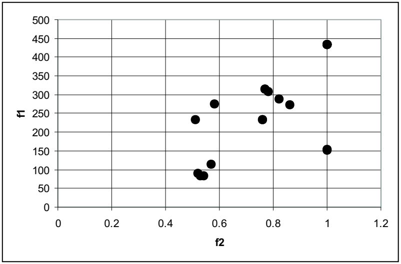

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Cardinality constraint on number of assets (Chang et al. 2000)

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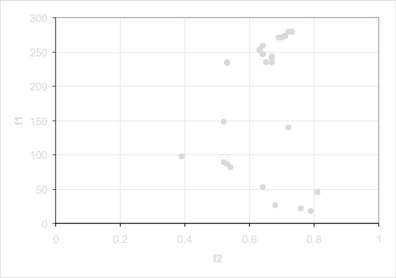

Nondominated points in five objective problem

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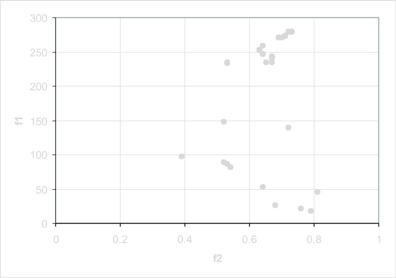

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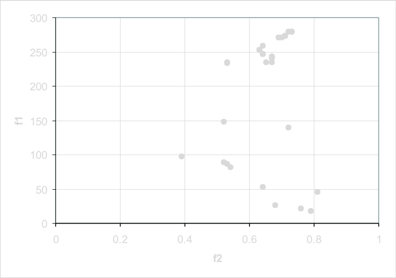

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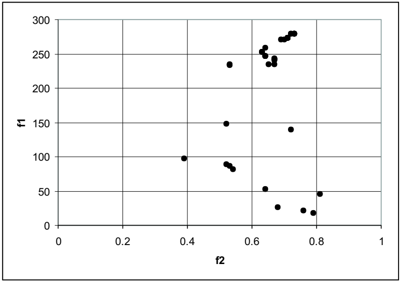

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Comments

- Multiobjective optimization provides explanation of phenomenon that has no explanation in standard framework
- Chapter 20 in Figueira, Greco, Ehrgott "Multicriteria Decision Analysis" Springer 2005

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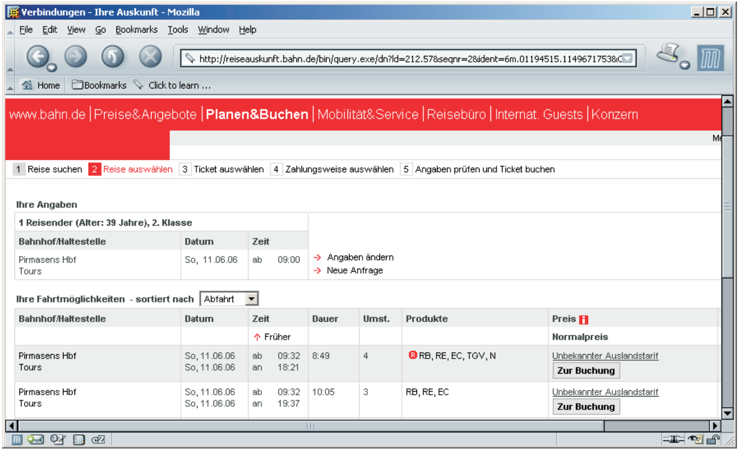
Overview

Online Timetable Information



Computation Times for Biobjective Shortest Path

- Known to be NP-hard and intractable
- Experiments by Raith 2007

Type	Nodes	Edges	Paths	CPU Time
Grid	4,902	19,596	6	<0.01
Grid	4,902	19,596	1,594	20.85
NetMaker	3,000	33,224	15	<0.01
NetMaker	14,000	153,742	17	0.02
Road	330,386	1,202,458	21	1.10
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Theoretical Results

Using ratio of first and second objective on arcs

Theorem (Müller-Hannemann and Weihe 2006)

- *Even if the ratio between first and second length of an arc assumes only 2 values there are exponentially many efficient paths.*
- *If k different ratios are allowed and the sequence of ratios switches only once between increasing and decreasing (bitonic path) then there are at most $O(n^{2k-2})$ efficient paths.*

Experimental Results

- 1.4 million nodes, 2.3 million arcs
- 84% of efficient paths are bitonic
- Distance versus time: average 2, maximum 8
- Fare versus time: average 3, maximum 22
- Distance, Time, Train changes: average 10, maximum 96

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- Worst case estimates may not apply in a particular application
- Problem size, structure and cost structure important
- Interesting mathematical results to be obtained from particular applications

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Airline Crew Scheduling

BBC NEWS

Sunday, 4 August, 2002, 20:29 GMT 21:29 UK

Delays as Easyjet cancels 19 flights

Passengers with low-cost airline Easyjet are suffering delays after 19 flights in and out of Britain were cancelled.

The company blamed the move - which comes a week after passengers staged a protest sit-in at Nice airport - on crewing problems stemming from technical hitches with aircraft.

Crews caught up in the delays worked up to their maximum hours and then had to be allowed home to rest.

Mobilising replacement crews has been a problem as it takes time to bring people to airports from home. Standby crews were already being used and other staff are on holiday.

Airline Crew Scheduling

Partition flights into set of pairings to minimize cost

$$a_{ij} = \begin{cases} 1 & \text{pairing } j \text{ includes flight } i \\ 0 & \text{otherwise} \end{cases}$$

$$\begin{aligned} \min &= c^T x \\ \text{subject to } Ax &= e \\ Mx &= b \\ x &\in \{0,1\}^n \end{aligned}$$

Big OR success story, used by all airlines

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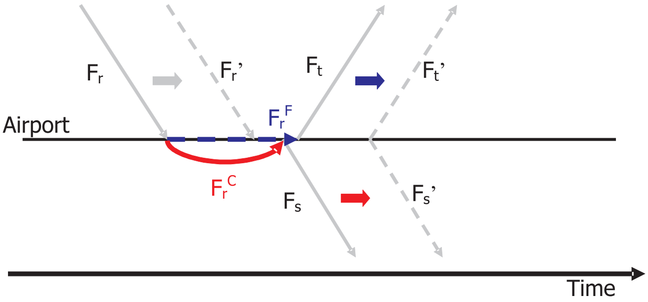

- Cost optimal solutions tend to minimize ground time
- Delays easily propagate through schedule

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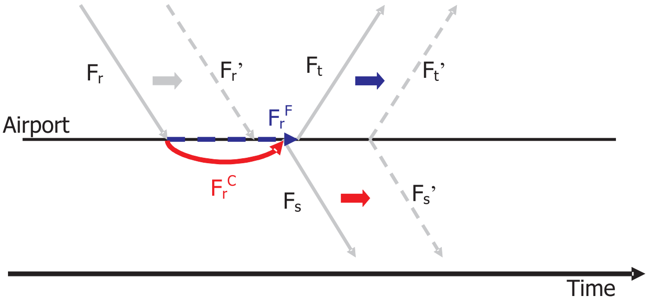

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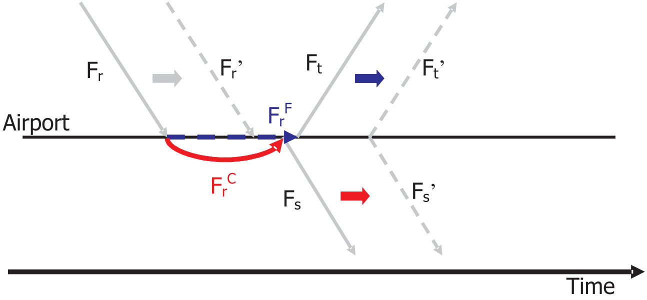

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Dealing with Delay

Solution 1: Stochastic programme with recourse (Yen and Birge 2006)

$$\begin{aligned} \min c^T x + Q(x) \\ \text{s.t. } Ax &= e \\ Mx &= b \\ x &\in \{0,1\} \end{aligned}$$

$Q(x) = \sum_{\omega \in \Omega} p(\omega) Q(x, \omega)$, where $Q(x, \omega)$ is delay under schedule x in scenario ω

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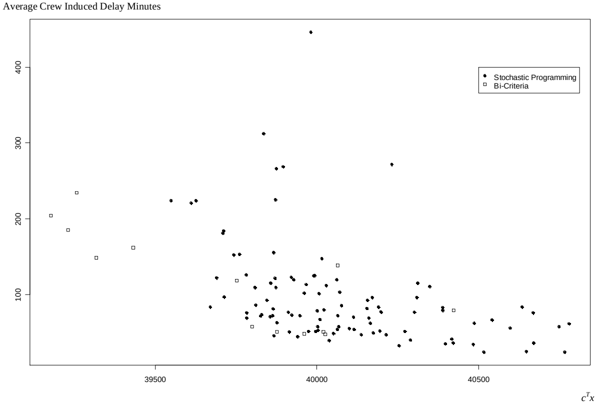
Simulation Results

Simulation of schedules obtained by both methods (Tam et al. 2007)



Solving Biobjective Set Partitioning Models

Method of elastic constraints

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\begin{aligned}
\min\ r^T x + ps \\
\text{s.t.}\quad Ax &= e \\
Mx &= b \\
c^T x - s + t &\leqq \varepsilon \\
x &\in \{0,1\}^n \\
s &\geqq 0
\end{aligned}
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- Solutions are weakly efficient
- All efficient solutions can be found
- Computationally superior

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Unit Crewing

What's the use of having robust solutions for pilots if cabin crew do something different?

- Solve pairings problem for several crew groups
- Minimize cost, maximize unit crewing (Tam et al. 2004)

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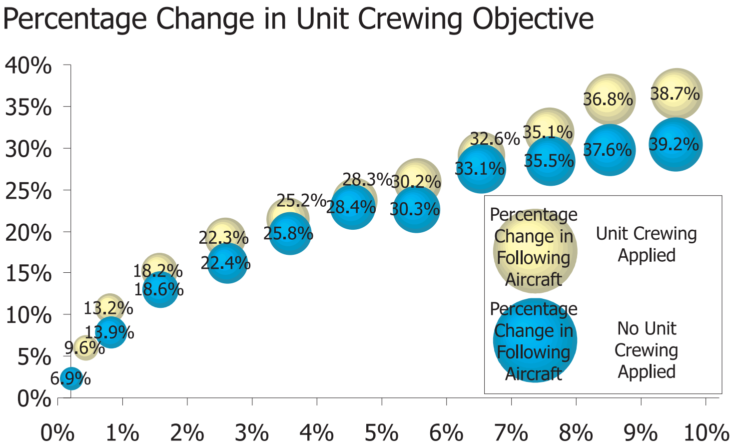
Results


Percentage Change in Unit Crewing Objective
40%
35%
30%
25%
20%
15%
10%
5%
0%
0%
1%
2%
3%
4%
5%
6%
7%
8%
9%
10%
9.6%
6.9%
13.2%
13.9%
18.2%
18.6%
22.3%
22.4%
25.2%
25.8%
28.3%
28.4%
30.2%
30.3%
32.6%
33.1%
35.1%
35.5%
36.8%
37.6%
38.7%
39.2%
Percentage Change in Following Aircraft
Unit Crewing Applied
Percentage Change in Following Aircraft
No Unit Crewing Applied

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- Biobjective model may be an alternative to stochastic programming, if recourse can be captured in deterministic objective
- Naturally drives development towards integrated model for airline operations (Weide et al. 2006)

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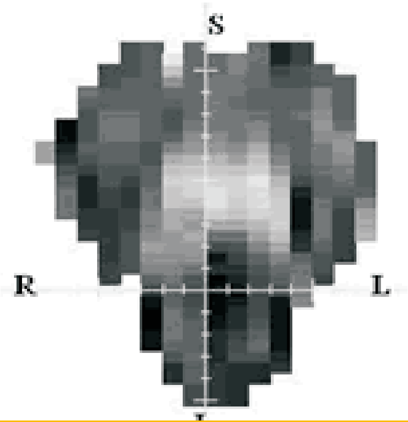
Overview

Radiotherapy

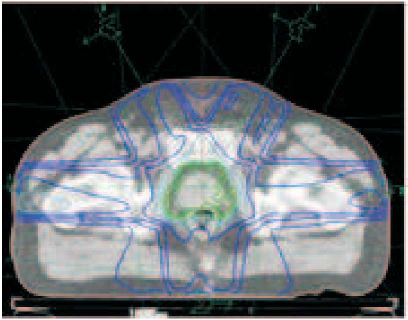
Radiotherapy

Radiotherapy Treatment Design

Given beam directions find intensity map



such that "good" dose distribution results



http://www.icr.ac.uk/ncri/liz_adams_rmt.html

Optimization Practice Today

(Intensity Modulated Radiotherapy) IMRT represents an advance in the means that radiation is delivered to the target, and it is believed that IMRT offers an improvement over conventional and conformal radiation in its ability to provide higher dose irradiation of tumor mass, while exposing the surrounding normal tissue to less radiation.
http://www.cancernews.com/data/Article/259.asp

- Most popular optimization model given goal dose to target, upper bounds for dose to critical structures and normal tissue

$$\min_{x \geqq 0} \omega_T \| A_{\mathcal{T}} x - TG \| + \omega_C \| (A_{\mathcal{C}} x - CG)_+ \| + \omega_N \| (A_{\mathcal{N}} x - NG)_+ \|$$

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Evolution of Multiobjective Models

- Standard dose based objective is weighted sum of multiobjective model
- But obvious MOP

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A Multiobjective LP Formulation

Theorem (Romeijn et al. 2004)

The MOPs $\min\{(f_1(x), \ldots, f_p(x)) : x \in X\}$ *and* $\min\{(h_1(f_1(x)), \ldots, h_p(f_p(x))) : x \in X\}$ *with strictly increasing* $h_1, \ldots, h_p$ *and convex* $f_1, \ldots, f_p$ *have the same efficient set.*

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\begin{array}{rcl}
\min(y_{\mathcal{T}}, y_{\mathcal{C}}, y_{\mathcal{N}}) & & \\
\text{subject to } A_{\mathcal{T}}x + y_{\mathcal{T}}e & \geqq & l_{\mathcal{T}} \\
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Results

- Only 3 objectives, so solve problem in objective space
- Benson's algorithm (Benson 1998)
- Dual variant of Benson's algorithm (Ehrgott, Loehne, Shao 2007)
- Dose matrix imprecise, delivery imprecise
- Calculation to small fraction of a Gy
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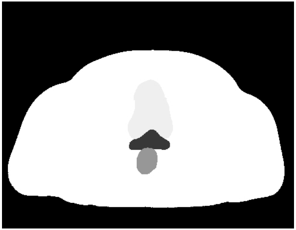
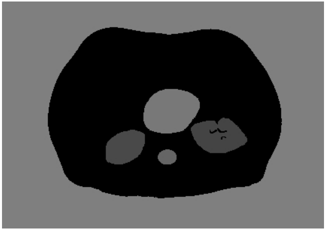
The Test Cases

Acoustic Neuroma



Prostate



Pancreatic Lesion

- Dose calculation inexact
- Inaccuracies during delivery
- Planning to small fraction of a Gy acceptable

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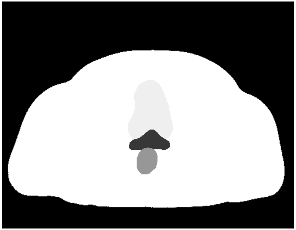

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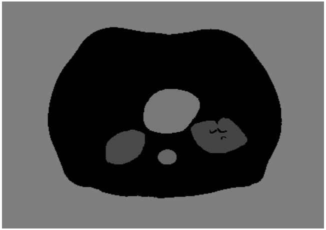

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The Test Cases

Case	AN	P	PL
Tumour voxels	9	22	67
Critical organ voxels	47	89	91
Normal tissue voxels	999	1182	986
Bixels	594	821	1140
$u_{\mathcal{T}}$	87.55	90.64	90.64
$l_{\mathcal{T}}$	82.45	85.36	85.36
$u_{\mathcal{C}}$	60/45	60/45	60/45
$u_{\mathcal{N}}$	0.00	0.00	0.00
$z_{\mathcal{T}}UB$	16.49	42.68	17.07
$z_{\mathcal{C}}UB$	12.00	30.00	12.00
$z_{\mathcal{N}}UB$	87.55	100.64	90.64

Numerical Results

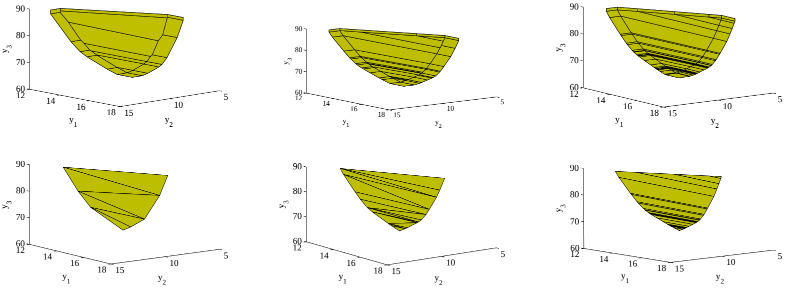
	ϵ	Solving the dual			Solving the primal		
		Time	Vert.	Cuts	Time	Vert.	Cuts
AC	0.1	1.484	17	8	5.938	27	21
	0.01	3.078	33	18	8.703	47	44
	0	8.864	85	55	13.984	55	85
PR	0.1	4.422	39	19	14.781	56	42
	0.01	18.454	157	78	64.954	296	184
	0	792.390	3280	3165	995.050	3165	3280
PL	0.1	58.263	85	44	164.360	152	90
	0.01	401.934	582	298	1184.950	1097	586
	0.005	734.784	1058	539	2147.530	1989	1041

Numerical Results

$\epsilon = 0.1$ $\epsilon = 0.01$ $\epsilon = 0$

P:

D:



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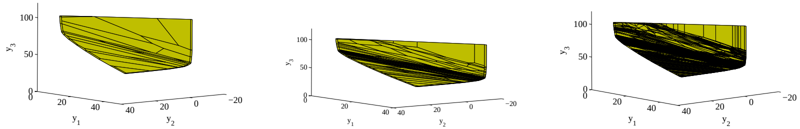

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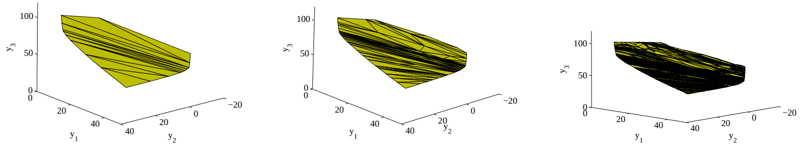

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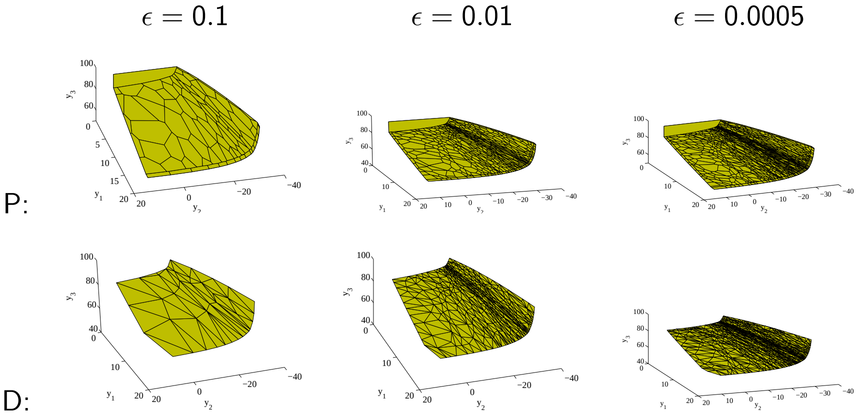

The Challenge

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x &\geqq 0 \\
x &\leqq My \\
e^T y &\leqq r \\
y &\in \{0,1\}^I
\end{aligned}
$$

where I is number of candidate beams

Comments

- It may be hard to convince practitioners of the usefulness of multiobjective optmization
- Exploit application to simplify methods
- Multiobjective optimization leads to improved processes
- New theoretical developments driven by application

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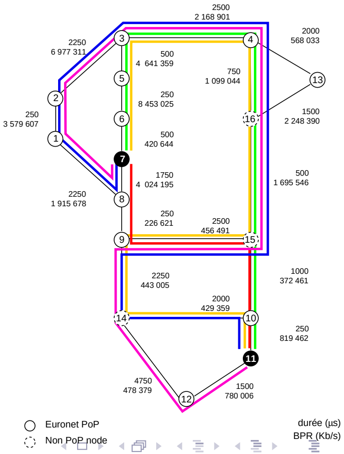
Overview

1. Metaheuristics
2. Finance
   - Portfolio Selection
3. Transportation
   - Train Timetable Information
   - Crew Scheduling
4. Medicine
   - Radiotherapy Treatment Design
5. Telecommunication
   - Routing in IP Networks
6. Conclusion

Routing in IP Networks

- Standard: OSPF protocol (open shortest path first)
- Dijkstra's algorithm used to minimize number of hops
- Other protocols allow aggregation of several objectives
- But still "best effort" rather than "Quality of Service"

EUROPE2



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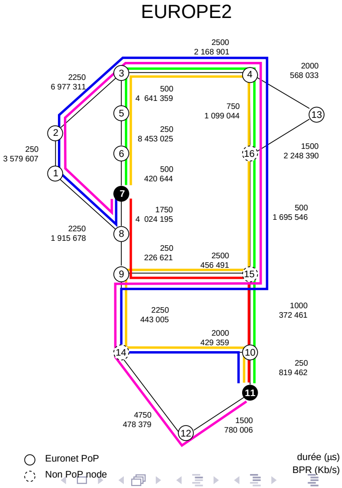

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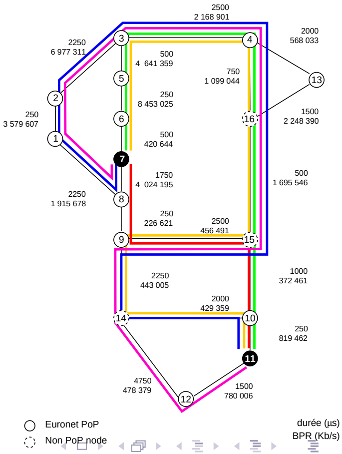

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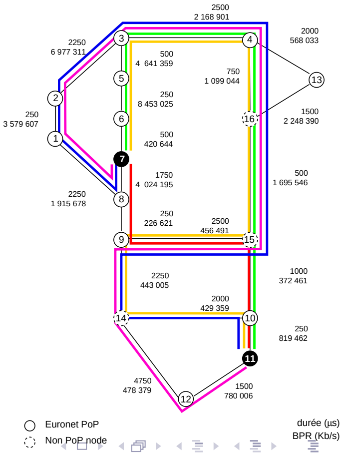

Multiobjective Shortest Paths

- Find paths (routes) p with objectives (Gandibleux et al. 2006)
    - $\min f_1(p) = \sum_{(i,j)\in p} c^1(i,j)$ (delay)
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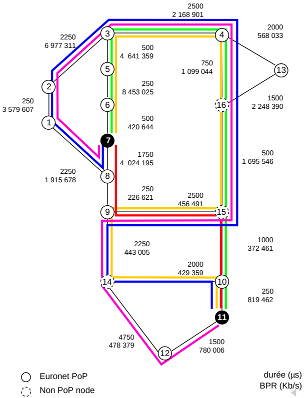
Result

With delay and bandwidth objectives there are 5 efficient paths from 7 to 11

EUROPE2



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- Chapter 22 in Figueira, Greco, Ehrgott "Multicriteria Decision Analysis" Springer 2005

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- Challenge 1: Two phase method for more than three objectives (Principle: Przybylski et al. 2007)
- Challenge 2: Multiobjective branch and bound algorithm (Spanjaard and Sourd 2007)
- Challenge 3: Polyhedral theory for MOCO scalarization
- Challenge 4: How to build an exact algorithm for very hard problems
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