Preference elicitation for Multiple Criteria Decision Aiding

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Algorithmic Decision Theory: MCDA and MOO

Contents

Introduction

- n criteria $g_1, g_2, \ldots, g_n$, $A = \{a_1, a_2, ..., a_m\}$ and Δ the dominance relation on A.
- preference information ($\mathcal{I}$)= any piece of information that can discriminate pairs of alternatives not in Δ,

$\longrightarrow$ Decision processus,

$\longrightarrow$ Decision aid process,

$\longrightarrow$ **Preference elicitation process**

Preference elicitation process

- $\mathcal{I} = \mathcal{I}^{in} \cup \mathcal{I}^{res}$,
- $\mathcal{I}^{in}$: input oriented preference information
  - "criterion g_3 is the most important one"
  - "the substitution rate between g_1 and g_4 is 3"
  - "The frontier between Cat_3 and Cat_2 on g_1 is equal to 12"
- $\mathcal{I}^{res}$: result oriented preference information result
  - "I prefer a_2 to a_7"
  - "a_{11} should be assigned at least to category C_3"
  - "I prefer a_2 à a_7 more than I prefer a_5 à a_1"

Preference elicitation process

- $\mathcal{P}$ an MCAP to which k preference parameters are attached $\overline{\upsilon} = (\upsilon_1, \upsilon_2, ...\upsilon_k)$,
- Ω the space of acceptable values for $\overline{\upsilon}$ in absence of preference information,
- The knowledge on $\overline{\upsilon}$ (stemming from $\mathcal{I}$) is defined by $\Omega(\mathcal{I}) \subseteq \Omega$ a list of constraints on $\overline{\upsilon}$,
- Specific case: $\Omega(\mathcal{I}) = \{\omega\}$
  $\hookrightarrow$ the value of preference parameter is fully determined,
- Otherwise, the value of at least one preference parameter is imprecisely known.

Preference elicitation process

- Applying an MCAP $\mathcal{P}$ to a subset of alternatives $A' \subseteq A$ using $\omega \in \Omega$, lead to a result $R_{\mathcal{P}}(A', \omega)$:

  Choice: a subset of selected alternatives $A^* \subseteq A'$
  Sorting: the assignment of each $a \in A'$ to a category
  Ranking: un partial preorder on A'

- Applying an MCAP $\mathcal{P}$ to a subset of alternatives $A' \subseteq A$ using $\Omega(\mathcal{I}) \subset \Omega$, lead to a result $R_{\mathcal{P}}(A', \Omega(\mathcal{I}))$,

- $R_{\mathcal{P}}(A', \Omega(\mathcal{I}))$ should account for each $\omega \in \Omega(\mathcal{I})$

Preference elicitation process

Given an MCAP $\mathcal{P}$ selected to model the DM's preferences, a **preference elicitation process** consists in an interaction between the DM and the analyst and leads the DM to express information on his/her preferences within the framework of $\mathcal{P}$.

Such information is materialized by a set $\Omega(\mathcal{I}) \subseteq \Omega$ of plausible values for the parameters of $\mathcal{P}$. At the end of the process, $\Omega(\mathcal{I})$ should lead, through the use of $\mathcal{P}$, to a result which is compatible with DM's view.

Preference elicitation process

- Preference elicitation process = element of the decision aiding process (stakeholder identification, definition of F and A),
- The definition is grounded on the prior selection of a MCAP,
- The notion of DM/analyst interaction is a constituent of the elicitation process (sequence of Q/A in which the DM progressively express preference information),
- During the elicitation process $\Omega(\mathcal{I}) \subseteq \Omega$ is defined progressively (by the sequence of Q/A),
- the obtained $\Omega(\mathcal{I}) \subseteq \Omega$ should lead, using $\mathcal{P}$, to a result consistent with the DM's view. Otherwise, the process should go on so as to revise $\Omega(\mathcal{I})$ consequently,

Nature of the preference elicitation activity

Two ways to consider the preference elicitation process

- $\longrightarrow$ the *descriptivist* approach,
- $\longrightarrow$ the *constructiviste* approach.

Preference elicitation : descriptivist approach

- The way alternatives compare is defined is the mind of the DM before the preference elicitation process starts,
- The elicitation process does not alter the pre-existing structure of preferences,
- Preference information is considered stable and refer to a reality,
- The preference model should account for the existing preferences as reliably as possible,
- There is a "*distinction between true and estimated weights and it is possible that subjects' true weights remain constant at all times, but become distorted in the elicitation process*". [Beattie et Barron 91]

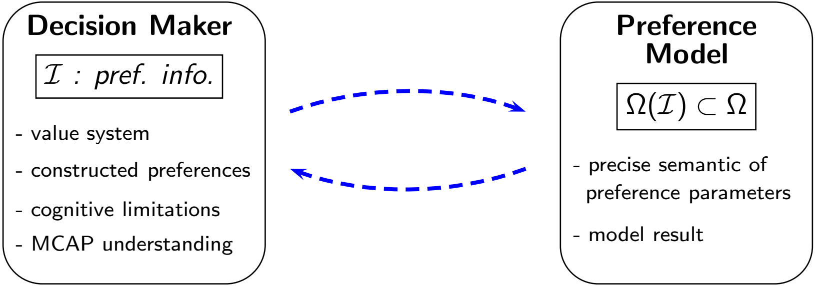
Preference elicitation: constructivist approach

- The constructivist approach considers preferences as not fully pre-established in the DM's mind,
- The purpose of preference elicitation is to specify and even to modify pre-existing elements,
- Parameters' values reflect, in the MCAP, statements expressed by the DM along the elicitation process.

Constructive learning preference elicitation

- Beyond the preference model elaboration, the elicitation process gives a concrete expression of DM's convictions about the way alternatives compare,
- Elaboration of such convictions are grounded on:
  - pre-existing elements such as his/her value system, past experience related to the decision problem, ...
  - the preference elicitation process itself.

Constructive learning preference elicitation



Preference elicitation tools for constructive learning

- Tools versus practice,
- Various "ingredients" can contribute to give birth to an Constructive Learning Preference Elicitation (CLPE) interaction,
  - aggregation / disaggregation (inference procedure),
  - elicitation and robustness,
  - inconsistency detection and resolution.

Disaggregation

Preference Information
$\mathcal{I}$

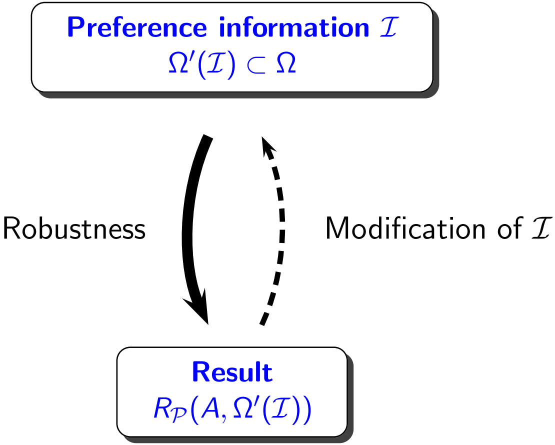
↓

Inference procedure

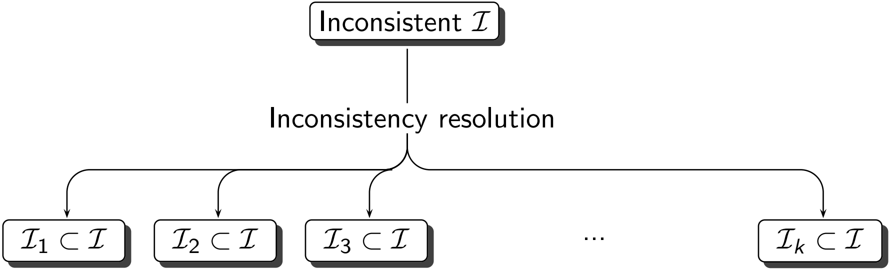
↓

inferred parameters: $\omega^*(\mathcal{I})$
$(\mathcal{P}, \omega^*(\mathcal{I}))$ = **preference model des préférences that "best" match** $\mathcal{I}$

Elicitation and Robustness



Inconsistency detection and resolution



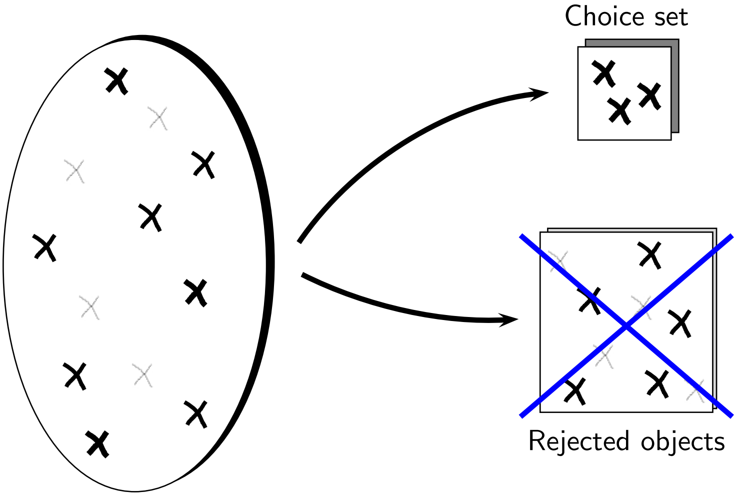
UTA-GMS

- Robust elicitation of a ranking model,
- Preference model = set of monotone additive value functions,
- Preference information = pairwise comparisons of alternatives/evaluation vectors and information about intensities of preference.

Problem statements

- Choosing, from a set of potential alternatives, the best alternative or a small sub set of the best alternatives
- Sorting alternatives to pre-defined and (ordered) categories
- Ranking the alternatives from the best to the worst (the ranking can be complete or not)

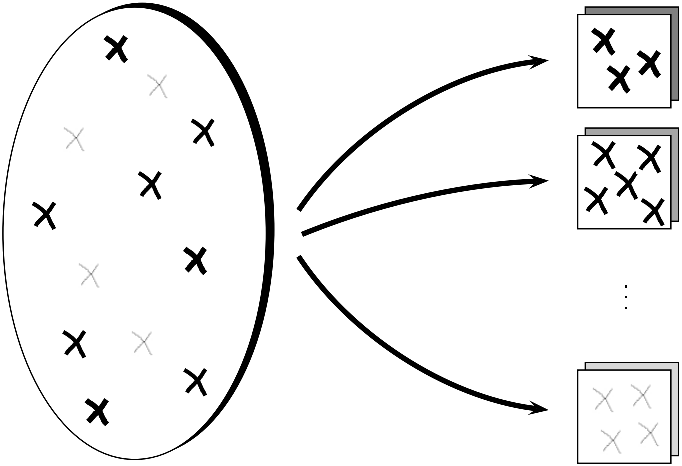
Choice problem statement



Problem statements

- Assigning alternatives to pre-defined and order categories

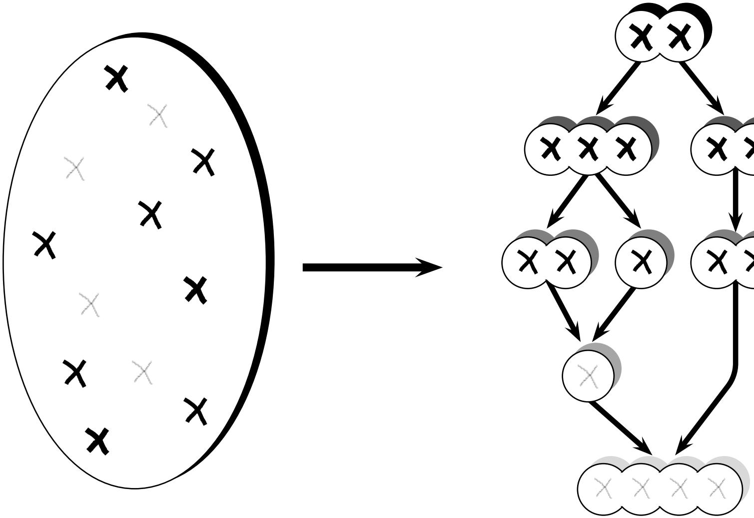
Sorting problem statement



Problem statements

- Ranking the alternatives from the best to the worst (the ranking can be complete or not)

Ranking problem statement



Ordinal regression paradigm

- Traditional aggregation paradigm: The criteria aggregation model is first constructed and then applied on set A to get information about the comprehensive preference
- Disaggregation-aggregation (or ordinal regression) paradigm: Comprehensive preferences on a subset $A^R \subset A$ is known a priori, and a consistent criteria aggregation model is inferred from this information to be applied on set A.

Ordinal regression paradigm

- In UTA^{GMS}, the preference model is a set of additive value functions compatible with a non-complete set of pairwise comparisons of reference alternatives and information about comprehensive and partial intensities of preference
- We focus on the ranking problem statement (but the ideas can be extended to choice and sorting)

Elementary notation

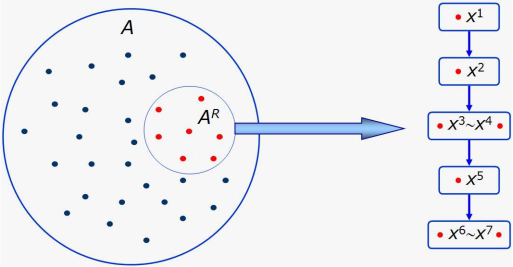
- $A = \{a_1, a_2, \ldots, a_i, \ldots, a_m\}$ is finite set of alternatives
- $g_1, g_2, \ldots, g_j, \ldots, g_n$ n criterion functions, F is the set of criteria indices
- $g_j(a_i)$ is the evaluation of the alternative a_i on criterion g_j
- G_j - domain of criterion g_j,
- $\succsim$ - weak preference (outranking) relation on G: for each $x, y \in G$
  - $x \succsim y \;\Leftrightarrow$ "x is at least as good as y"
  - $x \succ y \;\Leftrightarrow [x \succsim y$ and not$(y \succsim x)]$ "x is preferred to y"
  - $x \sim y \;\Leftrightarrow [x \succsim y$ and $y \succsim x]$ "x is indifferent to y"

Reminder on UTA

- For each g_j, $G_j = [\alpha_j, \beta_j]$ is the criterion evaluation scale, $\alpha_j \leq \beta_j$,
- U is an additive value function on G: for each $x \in G$, $U(x) = \sum_{j \in F} u_j[g_j(x)]$,
- u_j are non-decreasing marginal value functions, $u_j : G_j \mapsto \mathbb{R}$, $\forall j \in F$

Reminder on UTA

- The preference information is given in the form of a complete pre-order on a subset of reference alternatives $A^R \subseteq A$, called reference pre-order.
- $A^R = \{a_1, a_2, ..., a_{m1}\}$ is rearranged such that $a_k \succsim a_{k+1},\ k = 1, ..., m_1 - 1$, where $m_1 = |A^R|$.



Reminder on UTA

- The inferred value of each $a \in A^R$ is :

$$U(a) + \sigma^+(a) - \sigma^-(a),$$

- In UTA , the marginal value functions u_i are assumed to be piecewise linear, so that the intervals $[\alpha_i, \beta_i]$ are divided into $\gamma_i \geq 1$ equal sub-intervals

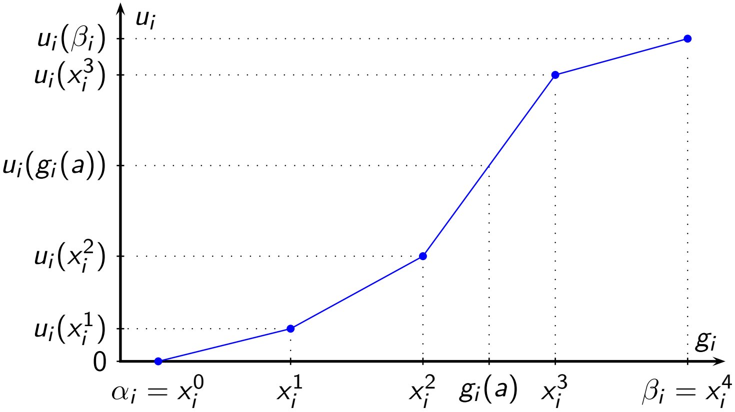
$$[x_i^0, x_i^1], [x_i^1, x_i^2], \ldots, [x_i^{\gamma_i - 1}, x_i^{\gamma_i}],$$

where,

$$x_i^j = \alpha_i + \frac{j(\beta_i - \alpha_i)}{\gamma_i}, j = 0, \ldots, \gamma_i, i = 1, \ldots, n.$$

Reminder on UTA

The piecewise linear value model is defined by the marginal values at break points: $u_i(x_i^0) = u_i(\alpha_i),\ u_i(x_i^1), u_i(x_i^2), \ldots, u_i(x_i^{\gamma_i}) = u_i(\beta_i)$



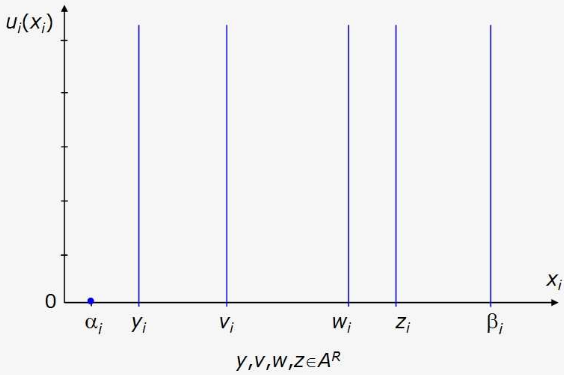
The UTA^{GMS} method: Main features

UTA^{GMS} method generalizes the UTA method in two aspects:

- It takes into account all additive value functions compatible with indirect preference information, while UTA is using only one such function.
- The marginal value functions are general monotone non-decreasing functions, and not piecewise linear only.

General monotone non-decreasing value functions

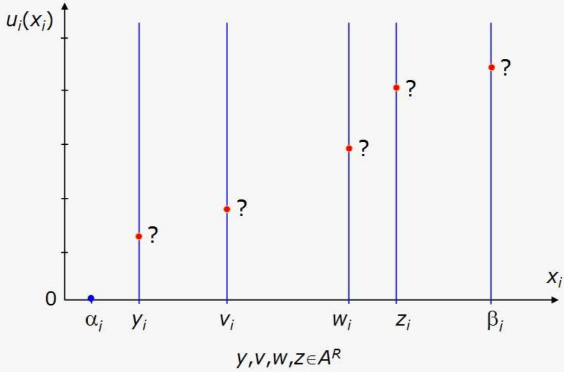
The marginal utility function $u_i(x_i)$



Characteristic points of marginal utility functions are fixed on actual evaluations of actions from A

General monotone non-decreasing value functions

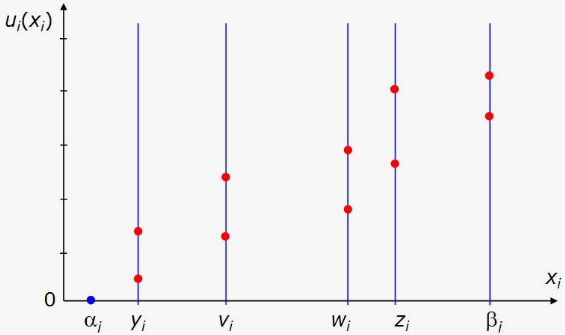
The marginal utility function $u_i(x_i)$



Marginal values in characteristic points are unknown

General monotone non-decreasing value functions

The marginal utility function $u_i(x_i)$

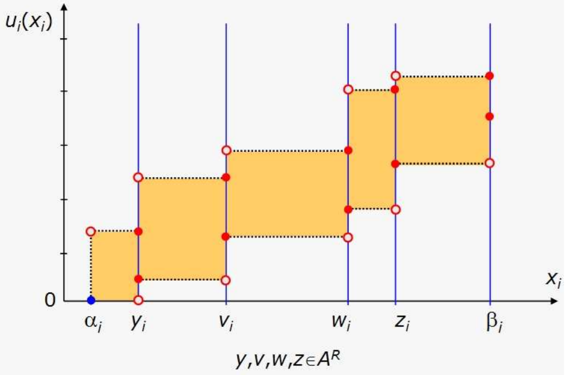


$y, v, w, z \in A^R$

In fact, they are intervals, because all compatible utility functions are considered

General monotone non-decreasing value functions

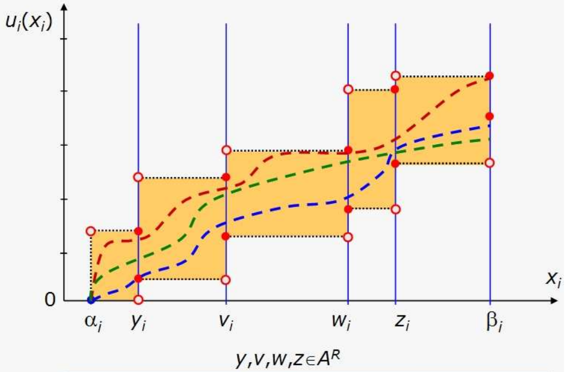
The marginal utility function $u_i(x_i)$



Area of all compatible marginal value functions

General monotone non-decreasing value functions

The marginal utility function $u_i(x_i)$

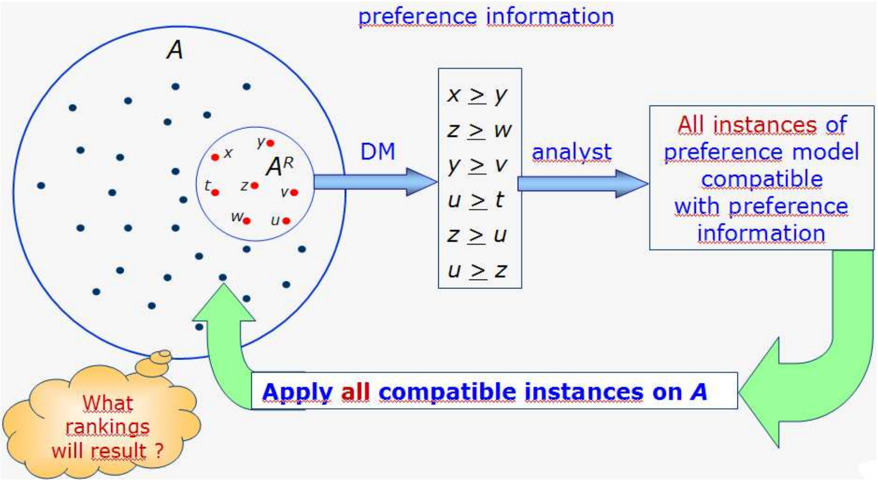


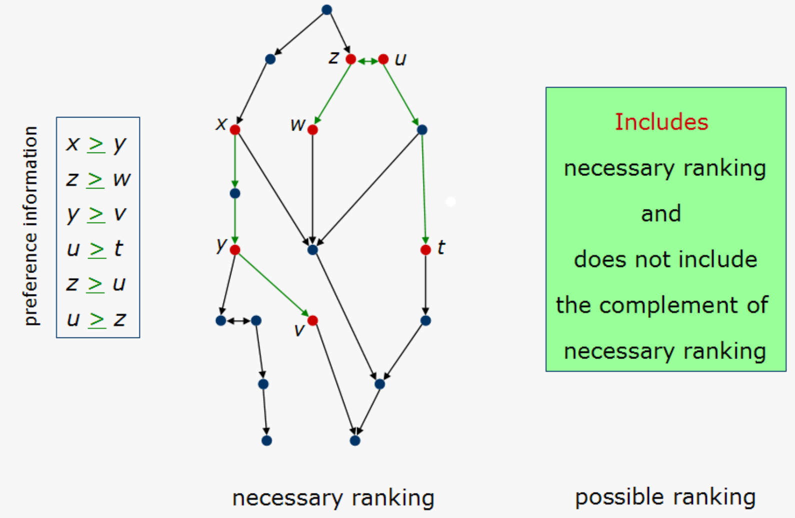
In the area the marginal compatible value functions must be monotone

The UTA^{GMS} method: Main features

The method produces two rankings in the set of alternatives A, such that for any pair of alternatives $a, b \in A$,

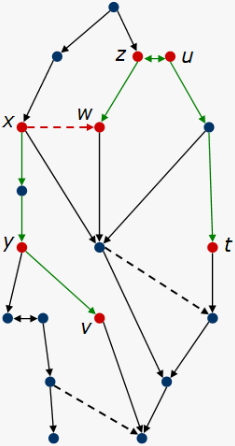
- In the *necessary order*, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for all value functions compatible with the preference information.
- In the *possible order*, a is ranked at least as good as b if and only if, $U(a) \geq U(b)$ for at least one value function compatible with the preference information.


preference information
A
x
y
A^R
t
z
v
w
u
DM
$x \geq y$
$z \geq w$
$y \geq v$
$u \geq t$
$z \geq u$
$u \geq z$
analyst
All instances of preference model compatible with preference information
Apply all compatible instances on A
What rankings will result ?


preference information
$x \succeq y$
$z \succeq w$
$y \succeq v$
$u \succeq t$
$z \succeq u$
$u \succeq z$
z
u
x
w
y
t
v
Includes
necessary ranking
and
does not include
the complement of
necessary ranking
necessary ranking
possible ranking

additional preference information

$x \succsim y$
$z \succsim w$
$y \succsim v$
$u \succsim t$
$z \succsim u$
$u \succsim z$
$x \succsim w$



Includes

necessary ranking

and

does not include

the complement of

necessary ranking

necessary ranking
enriched

possible ranking
impoverished

Computing necessary and possible relations ($\succsim^N$ and $\succsim^P$)

- Let d(x,y)= $Min_{U \in \mathcal{U}} U(x) - U(y)$ and
  D(x,y)= $Max_{U \in \mathcal{U}} U(x) - U(y)$
  where $\mathcal{U} = \{$value fonctions compatible with the DM's statements$\}$
- $x \succsim^N y \;\Leftrightarrow\; d(x,y) \geq 0$
- $x \succsim^P y \;\Leftrightarrow\; D(x,y) \geq 0$
- Properties:
  - $x \succsim^N y \Rightarrow x \succsim^P y$,
  - $\succsim^N$ is a partial preorder (reflexive and transitive),
  - $\succsim^P$ is strongly complete ($x \succsim^P y$ or $y \succsim^P x$), but not necessarily transitive.

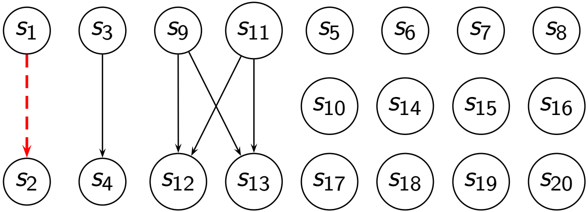
Illustrative example

20 alternatives, 5 criteria (all alternatives are efficient).

s_1	$= (14.5, 147, 4, 1014, 5.25)$	s_{11}	$= (15.75, 164.375, 41.5, 311, 6.5)$
s_2	$= (13.25, 199.125, 4, 1014, 4)$	s_{12}	$= (13.25, 181.75, 41.5, 311, 4)$
s_3	$= (15.75, 164.375, 16.5, 838.25, 5.25)$	s_{13}	$= (12, 199.125, 41.5, 311, 2.75)$
s_4	$= (12, 181.75, 16.5, 838.25, 4)$	s_{14}	$= (17, 147, 16.5, 662.5, 5.25)$
s_5	$= (12, 164.375, 54, 838.25, 4)$	s_{15}	$= (15.75, 199.125, 16.5, 311, 6.5)$
s_6	$= (13.25, 199.125, 29, 662.5, 5.25)$	s_{16}	$= (13.25, 164.375, 54, 311, 4)$
s_7	$= (13.25, 147, 41.5, 662.5, 5.25)$	s_{17}	$= (17, 181.75, 16.5, 486.75, 5.25)$
s_8	$= (17, 216.5, 16.5, 486.75, 1.5)$	s_{18}	$= (14.5, 164.375, 41.5, 838.25, 4)$
s_9	$= (17, 147, 41.5, 486.75, 5.25)$	s_{19}	$= (15.75, 181.75, 41.5, 135.25, 5.25)$
s_{10}	$= (15.75, 216.5, 41.5, 662.5, 1.5)$	s_{20}	$= (15.75, 181.75, 41.5, 311, 2.75)$

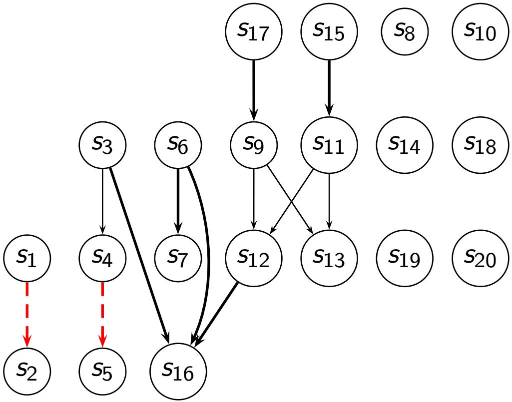
Illustrative example

First information: $s_1 \succ s_2$.



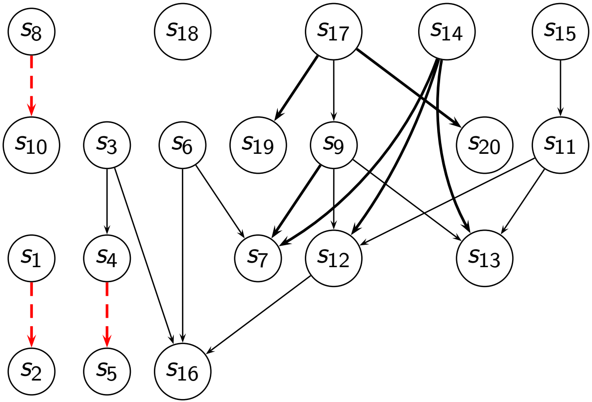
Illustrative example

Second information: $s_4 \succ s_5$.



Illustrative example

Third information: $s_8 \succ s_{10}$.



Inconsistency management

- When DM's statement are not representable in the additive model
  →inconsistency,
- DM's statements induce linear constraints on the variables (marginal values of alternatives)
- When such inconsistency occurs, we should check how to "solve" inconsistency,
- Which modification of the DM's input will lead to representable preferences ?
- Are they different ways to do so ?
- What is the minimum number of constraints to delete ?

Inconsistency management

- solution of minimal cardinality is not necessarily the most interesting one for the DM,
- The knowledge of the various ways to solve inconsistency is useful for the DM,
- This permits to:
  - help the DM to understand the conflictual aspect of his/her statement,
  - create a context in which the DM car learn about his/her preferences,
  - make the elicitation process more flexible,

Inconsistency resolution via constraints deletion

- m contraintes induced by the DM's statements

$$\left\{\begin{array}{lll} \sum_{j=1}^{n} \alpha_1^j x_j & \geq & \beta_1 \\ & \vdots & \\ \sum_{j=1}^{n} \alpha_{m-1}^j x_j & \geq & \beta_{m-1} \\ \sum_{j=1}^{n} \alpha_m^j x_j & \geq & \beta_m \end{array}\right. \quad [1]$$

- $I = \{1, \ldots, m\}$; subset $S \subset I$ resolves [1] iff $I \setminus S \neq \emptyset$
- We search for $S_1, S_2, \ldots, S_p \subset I$ such that :
  - (i) S_i resolves [1], $i \in \{1, 2, ..., p\}$;
  - (ii) $S_i \nsubseteq S_j, i, j \in \{1, ..., p\}, i \neq j$;
  - (iii) $|S_i| \leq |S_j|, i, j \in \{1, 2, ..., p\}, i < j$;
  - (iv) if $\exists$ S that resolves [1] s.t. $S \nsubseteq S_i$, $\forall i = 1, 2, \ldots, p$, then $|S| > |S_p|$.

Inconsistency management

- Soit y_i $(\in \{0,1\},\ i \in I)$, t.q. :

$$\begin{array}{rcl} y_i & = & 1 \text{ if constraint } i \text{ is removed} \\ & = & 0 \text{ otherwise} \end{array}$$

$$P_1 \left\{ \begin{array}{ll} Min & \sum_{i \in I} y_i \\ \text{s.t.} & \sum_{j=1}^{n} \alpha_{ij} x_j + \alpha'_i \lambda + M y_i \geq \beta_i, \ \ \forall i \in I \\ & x_j \geq 0, \ \ j = 1, \ldots, n \\ & y_i \in \{0,1\}, \ \ \forall i \in I \end{array} \right.$$

- $S_1 = \{i \in I : y_i^* = 1\}$ corresponds to (one of the) subset(s) of constraints resolving [1] of smallest cardinality,
- We define P_2 adding to P_1 the constraint $\sum_{i \in S_1} y_i \leq |S_1| - 1$

Inconsistency management

- P_{k+1} is defined adding to P_k the constraint $\sum_{i \in S_k} y_i \leq |S_k| - 1$
- We compute $S_1, S_2, \ldots, S_k$, and stop when $|S_{k+1}| > \Omega$,

```
Begin
    k ⟵ 1
    moresol ⟵ true
    While moresol
      Solve PM_k
      If (PM_k has no solution) or (PM_k has an optimal value > Ω)
        Then moresol ⟵ false
        Else
          - S_k ← {i ∈ I : y_i^* = 1}
          - Add constraint ∑_{i∈S_k} y_i ≤ |S_k| − 1 to PM_k so as to define PM_{k+1}
          - k ⟵k+1
      End if
    End while
End
```

Inconsistency management

- Each S_i corresponds to a set of DM's preference statements (presented to the DM),
- Sets S_i represent (for the DM) "incompatible" comparisons, each one specifies a way to solve inconsistency.

The GRIP method: Main features

GRIP extends UTA^{GMS} method by taking into account additional preference information in form of comparisons of intensities of preference between some pairs of reference alternatives. For

alternatives $x, y, w, z \in A$, these comparisons are expressed in two possible ways (not exclusive),

1) Comprehensively, on all criteria, "x is preferred to y at least as much as w is preferred to z".

2) Partially, on each criterion, "x is preferred to y at least as much as w is preferred to z, on criterion $g_i \in F$".

The GRIP method: Preference Information

DM is expected to provide the following preference information,

- A partial pre-order $\succsim$ on A^R whose meaning is: for $x, y \in A^R$

$$x \succsim y \Leftrightarrow x \text{ is at least as good as } y.$$

- A partial pre-order $\succsim^*$ on $A^R \times A^R$, whose meaning is: for $x, y, w, z \in A^R$,

$$(x, y) \succsim^* (w, z) \Leftrightarrow \ x \text{ is preferred to } y \text{ at least as much as } w \ .$$

is preferred to z

- A partial pre-order $\succsim_i^*$ on $A^R \times A^R$, whose meaning is: for $x, y, w, z \in A^R$,

$$(x, y) \succsim_i^* (w, z) \Leftrightarrow \ x \text{ is preferred to } y \text{ at least as much as } w$$

is preferred to z on criterion g_i, $i \in I$.

Software demonstration

Software demonstration: Visual-UTA 2.0

- AGRITEC is a medium size firm (350 persons approx.) producing some tools for agriculture,
- The C.E.O., M^{r} Becault, intends to double the production and multiply exports by 4 within 5 years.
- He wants to hire a new international sales manager.
- A recruitment agency has interviewed 17 potential candidates which have been evaluated on 3 criteria (sales management experience, international experience, human qualities) evaluated on a [0,100] scale.

Software demonstration

	Crit 1	Crit 2	Crit 3
Alexievich	4	16	63
Bassama	28	18	28
Calvet	26	40	44
Dubois	2	2	68
El Mrabat	18	17	14
Ferret	35	62	25
Fleichman	7	55	12
Fourny	25	30	12
Frechet	9	62	88
Martin	0	24	73
Petron	6	15	100
Psorgos	16	9	0
Smith	26	17	17
Varlot	62	43	0
Yu	1	32	64

Software demonstration

- it0 without preference information,
- it1 Ferret $\sim$ Frechet $\succ$ Fourny $\succ$ Fleichman,
- it2 Ferret $\sim$ Frechet $\succ$ Martin $\succ$ Fourny $\sim$ El Mrabat $\succ$ Fleichman,
  $\rightarrow$inconsistency: Ferret $\sim$ Frechet vs Fourny $\sim$ El Mrabat
- it3 Ferret $\sim$ Frechet $\succ$ Martin $\succ$ Fourny $\succ$ Fleichman,

Conclusion

- More work should be devoted to preference elicitation in MCDA,
- UTA-GMS:
  - General additive value function,
  - Intuitive information required from the DM,
  - Robust elicitation of a ranking model,
  - Necessary and Possible rankings,
  - Inconsistency management.

Conclusion

Unsufficient attention is devoted in MCDA to develop elicitation tools an methodologies which should contribute to the definition of a doctrine for MCDA practitioners.

More research is needed to :

- develop methodologies/tools to organize the interaction with DMs in a given MCAP,
- test the operational validity of the developed tools.