Preference elicitation for MCDA
An introduction

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Contents

Contents

Basic MCDA concepts

- Decision process,
- Stakeholders, actors, decision makers,
- Set of alternatives,
- Problems statements,

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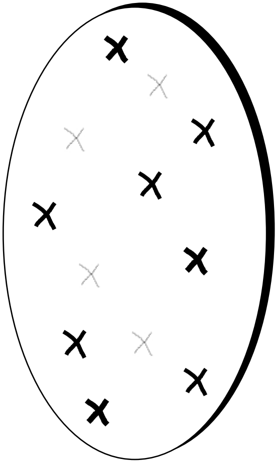
- Problems statements,

Problem statements

- Choosing, in a set of alternatives, the best (or a limited number of) alternatives(s)
- Ranking alternatives from the best to the worst (ranking can be complete or not)
- Sorting alternatives to pre-defined categories

Choice problem statement



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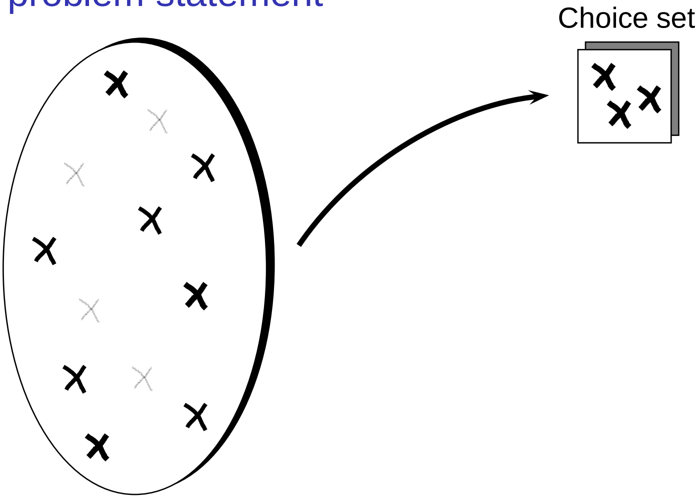

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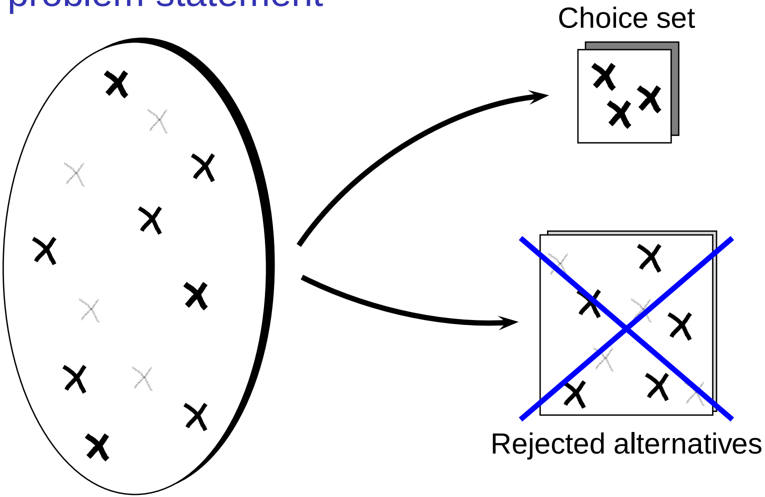
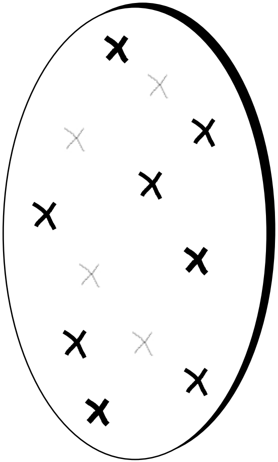


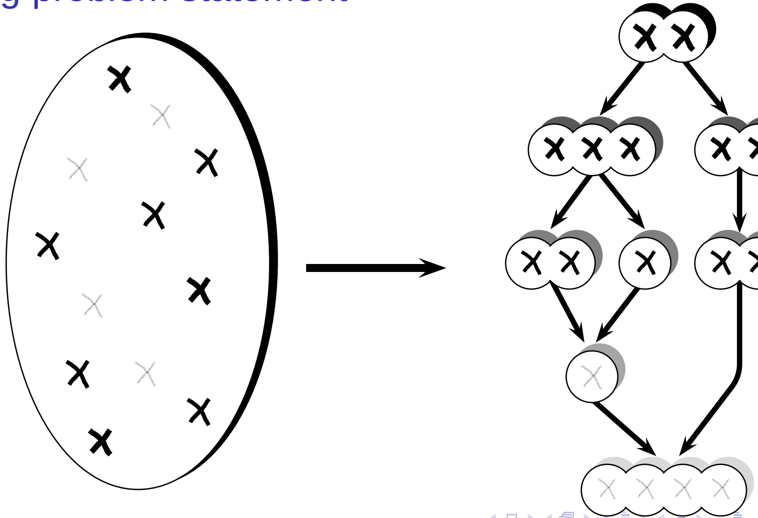
Problem statements

- Rank alternatives from the best to the worst (ranking can be complete or not)

Ranking problem statement



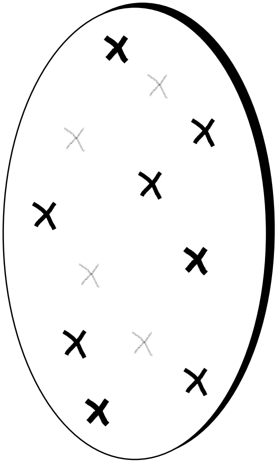
Ranking problem statement



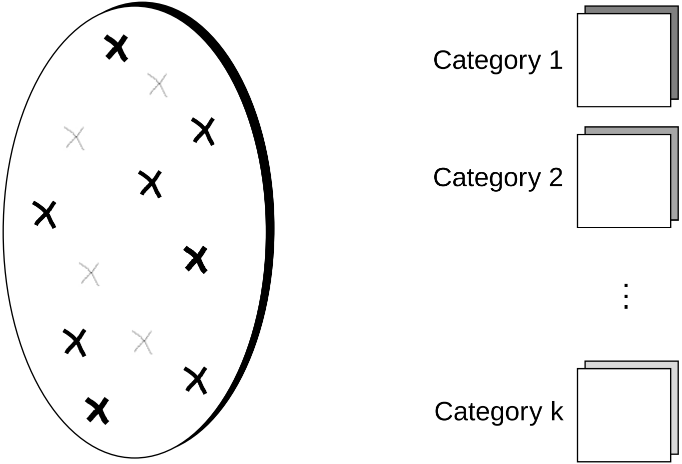
Problem statements

- Assign alternatives to pre-defined categories

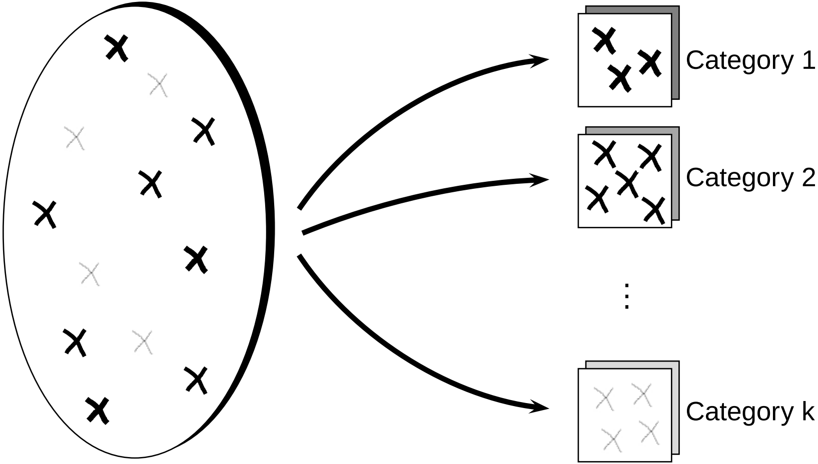
Sorting problem statement



Sorting problem statement



Sorting problem statement



Basic MCDA concepts

- Decision process,
- Stakeholders, actors, decision makers,
- Set of alternatives,
- Problems statements,
- Criterion,

Concept of criterion

- A criterion is a real valued function g defined on A allowing to compare any pair of alternatives according to a dimension (*axe de signification*) s.t.:

$$g(a) > g(b) \;\Rightarrow\; aS_g b,\; \forall a, b \in A$$

- Let us denote $F = \{1, 2, ..., n_{crit}\}$

Basic MCDA concepts

- Decision process,
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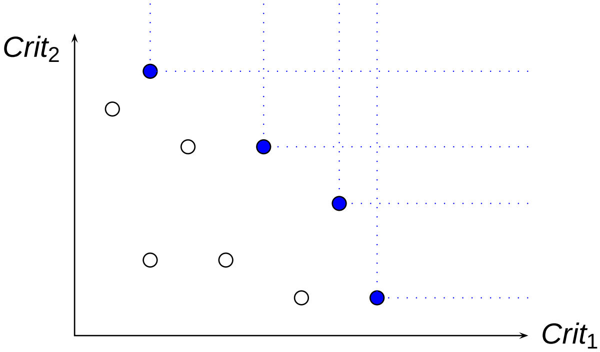
- Dominance, Pareto-optimality,

Dominance, Pareto-optimality

- $\forall a, b \in A$, $a \Delta b$ iff $g_j(a) \geq g_j(b)$, $\forall j \in F$, one of the inequalities being strict,
- The dominance relation Δ expresses unanimity among criteria in favor of one action in the comparison,
- Δ defines on A strict partial order (asymmetric and transitive),
- Δ is usually very poor,
- $a \in A$ is Pareto-optimal iff $\nexists b \in A$ s.t. $b \Delta a$,

Pareto front



Pareto front in a discret bi-criterion problem

Basic MCDA concepts

- Decision process,
- Stakeholders, actors, decision makers,
- Set of alternatives,
- Problems statements,
- Criterion,
- Dominance, Pareto-optimality,
- Preference information,

Preference information

- To discriminate among Pareto-optimal alternatives, the dominance relation Δ is useless,
- Decision aiding requires to enrich Δ by additional information additionnelle called **preference information**,
- Preference information refers to the DM's opinions, value system, convictions ... concerning the decision problem,
- It is standard to distinguish:
  - **Intracriterion** preference information, and
  - **Intercritria** preference information.

Basic MCDA concepts

- Decision process,
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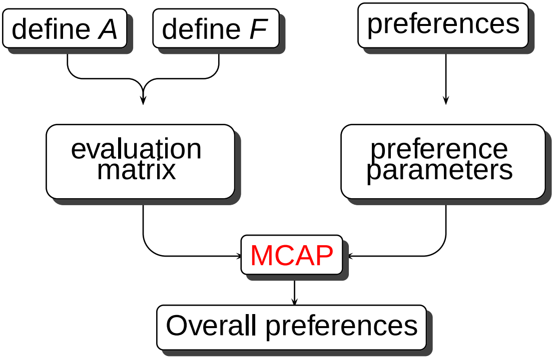
- Criterion,
- Dominance, Pareto-optimality,
- Preference information,
- Aggregation procedures,

Multiple criteria aggregation procedures (MCAP)

MCAP: Procedure which establishes overall preferences on A, based on preference information and the evaluation matrix.



Contents

Introduction

- n criteria $g_1, g_2, \ldots, g_n$, $A = \{a_1, a_2, ..., a_m\}$ and Δ the dominance relation on A.
- preference information ($\mathcal{I}$)= any piece of information that can discriminate pairs of alternatives not in Δ,

$\longrightarrow$ Decision process,

$\longrightarrow$ Decision aid process,

$\longrightarrow$ **Preference elicitation process**

Preference elicitation process

- $\mathcal{I} = \mathcal{I}^{in} \cup \mathcal{I}^{res}$,
- $\mathcal{I}^{in}$: input oriented preference information
  - “criterion g_3 is the most important one”
  - “the substitution rate between g_1 and g_4 is 3”
  - “The frontier between Cat_3 and Cat_2 on g_1 is equal to 12”
- $\mathcal{I}^{res}$: result oriented preference information result
  - “I prefer a_2 to a_7”
  - “a_{11} should be assigned at least to category C_3”
  - “I prefer a_2 to a_7 more than I prefer a_5 to a_1”

Preference parameters' values

"Values for preference parameters are meaningless as long as the MCAP in which they are used is not specified"

- Consider two alternatives $a = (15, 10, 10)$ and $b = (10, 12, 12)$.
- Suppose the criteria weights are equal $w = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$,
- Considering the weighted sum, $g(a) = 11.66 < g(b) = 11.33 \Rightarrow aPb$,
- In the Condorcet aggregation, as $\sum_{j:a \succsim_j b} w_j > \sum_{j:b \succsim_j a} w_j \Rightarrow bPa$,

Preference elicitation process

- $\mathcal{P}$ an MCAP to which k preference parameters are attached $\overline{\upsilon} = (\upsilon_1, \upsilon_2, ...\upsilon_k)$,
- Ω the space of acceptable values for $\overline{\upsilon}$ in absence of preference information,
- The knowledge on $\overline{\upsilon}$ (stemming from $\mathcal{I}$) is defined by $\Omega(\mathcal{I}) \subseteq \Omega$ a list of constraints on $\overline{\upsilon}$,
- Specific case: $\Omega(\mathcal{I}) = \{\omega\}$
  $\hookrightarrow$ the value of preference parameter is fully determined,
- Otherwise, the value of at least one preference parameter is imprecisely known.

Preference elicitation process

Illustration of the notations

- $\mathcal{P}$ = weighted sum, $\overline{w}$ = weights = $(w_1, w_2, ..., w_n)$,
- $\Omega = \{w_i \geq 0, i \in F : \sum_{i \in F} w_i = 1\}$,
- $\mathcal{I}$ induces a knowledge on $\overline{v}$: $\Omega(\mathcal{I}) \subseteq \Omega$, e.g. , if $\mathcal{I}$ = $\{(11, 10, 10) \succsim (10, 12, 10)\}$, then $\Omega(\mathcal{I}) = \{\overline{w} \in \Omega : w_1 \geq 2w_2\}$
- Specific case: $\Omega(\mathcal{I}) = \{w\}$, e.g. when (10,10,11)$\sim$(10,11,10)$\sim$(11,10,10).
- Otherwise, the value of at least one preference parameter is imprecisely known.

Preference elicitation process : an example

MCAP: weighted sum $g(a) = \sum_{i=1}^{n_{crit}} w_i.g_i(a)$

1. Consider b_j such that $g_i(b_j) = g_i^{min}$, $\forall i \neq j$ and $g_j(b_j) = u_j^{max}$.
   Rank the b_j, $j \in F$ (suppose $b_n \succ \ldots \succ b_1$),
   We get $w_n \geq \ldots \geq w_1$
2. Define b_n^j the alternative s.t. $g_i(b_n^j) = g_i^{min}, \forall i \neq n$;
   Determine $g_n(b_n^j)$ s.t. $b_1 I b_n^j$
   then $u(b_n^j) = u(b_1)$ hence
   $\sum_{i=1}^{n} u_i(b_1) = \sum_{i=1}^{n} u_i(b_n^j)$
   $100.w_1 = u_n(g_n(b_n^j)).w_1$, therefore $\frac{w_n}{w_1} = \frac{100}{u_i(x_i)}$
3. proceed simultaneously for g_2, ..., g_{n-1} to define the ratios $\frac{w_n}{w_i}$, $i = 1, \ldots, n-1$,

Preference elicitation process

- Applying an MCAP $\mathcal{P}$ to a subset of alternatives $A' \subseteq A$ using $\omega \in \Omega$, lead to a result $R_{\mathcal{P}}(A', \omega)$:
  - Choice: a subset of selected alternatives $A^* \subseteq A'$
  - Sorting: the assignment of each $a \in A'$ to a category
  - Ranking: un partial preorder on A'
- Applying an MCAP $\mathcal{P}$ to a subset of alternatives $A' \subseteq A$ using $\Omega(\mathcal{I}) \subset \Omega$, lead to a result $R_{\mathcal{P}}(A', \Omega(\mathcal{I}))$,
- $R_{\mathcal{P}}(A', \Omega(\mathcal{I}))$ should account for each $\omega \in \Omega(\mathcal{I})$

Preference elicitation process

Given an MCAP $\mathcal{P}$ selected to model the DM's preferences, a **preference elicitation process** consists in an interaction between the DM and the analyst and leads the DM to express information on his/her preferences within the framework of $\mathcal{P}$.

Such information is materialized by a set $\Omega(\mathcal{I}) \subseteq \Omega$ of plausible values for the parameters of $\mathcal{P}$. At the end of the process, $\Omega(\mathcal{I})$ should lead, through the use of $\mathcal{P}$, to a result which is compatible with DM's view.

Preference elicitation process

- Preference elicitation process $\subset$ decision aiding process (stakeholder identification, definition of F and A),
- The definition is grounded on the prior selection of a MCAP,
- The notion of DM/analyst interaction is a constituent of the elicitation process (sequence of Q/A in which the DM progressively express preference information),
- During the elicitation process $\Omega(\mathcal{I}) \subseteq \Omega$ is defined progressively (by the sequence of Q/A),
- the obtained $\Omega(\mathcal{I}) \subseteq \Omega$ should lead, using $\mathcal{P}$, to a result consistent with the DM's view. Otherwise, the process should go on so as to revise $\Omega(\mathcal{I})$ consequently,

Contents

Nature of the preference elicitation activity

Two ways to consider the preference elicitation process

- $\longrightarrow$ the *descriptivist* approach,
- $\longrightarrow$ the *constructivist* approach.

Preference elicitation : descriptivist approach

- The way alternatives compare is defined is the mind of the DM before the preference elicitation process starts,
- The elicitation process does not alter the pre-existing structure of preferences,
- Preference information is considered stable and refer to a reality,
- The preference model should account for the existing preferences as reliably as possible,
- There is a "*distinction between true and estimated weights and it is possible that subjects' true weights remain constant at all times, but become distorted in the elicitation process*". [Beattie et Barron 91]

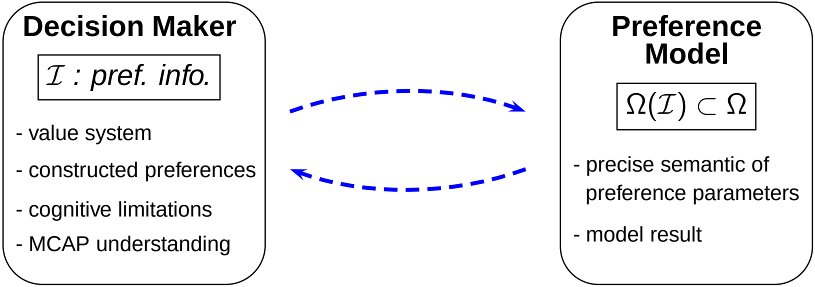
Preference elicitation: constructivist approach

- The constructivist approach considers preferences as not fully pre-established in the DM's mind,
- The purpose of preference elicitation is to specify and even to modify pre-existing elements,
- Parameters' values reflect, in the MCAP, statements expressed by the DM along the elicitation process.

Constructive learning preference elicitation

- Beyond the preference model elaboration, the elicitation process gives a concrete expression of DM's convictions about the way alternatives compare,
- Elaboration of such convictions are grounded on:
  - pre-existing elements such as his/her value system, past experience related to the decision problem, ...
  - the preference elicitation process itself.

Constructive learning preference elicitation

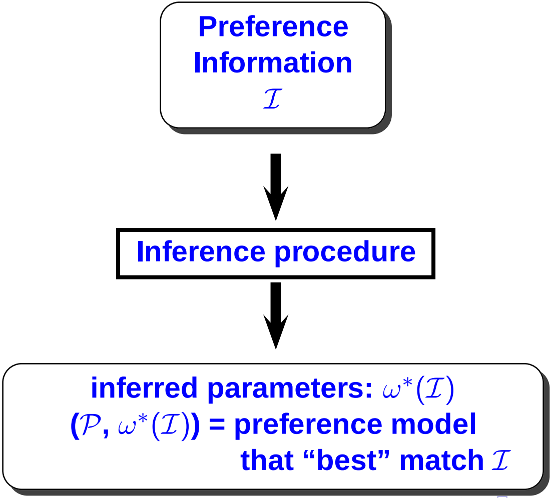


Contents

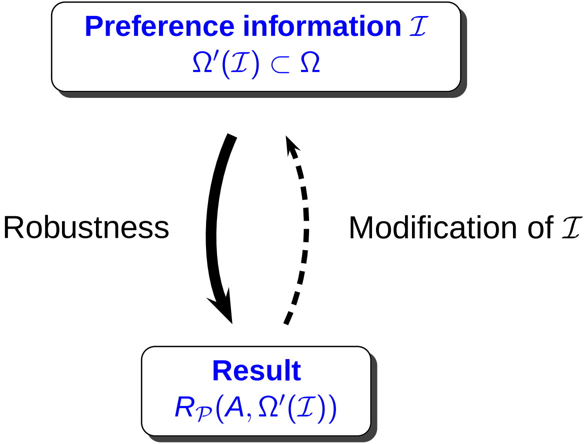
Preference elicitation tools for constructive learning

- Tools versus practice,
- Various “ingredients” can contribute to give birth to an Constructive Learning Preference Elicitation (CLPE) interaction,
    - aggregation / disaggregation (inference procedure),
    - elicitation and robustness,
    - inconsistency detection and resolution.

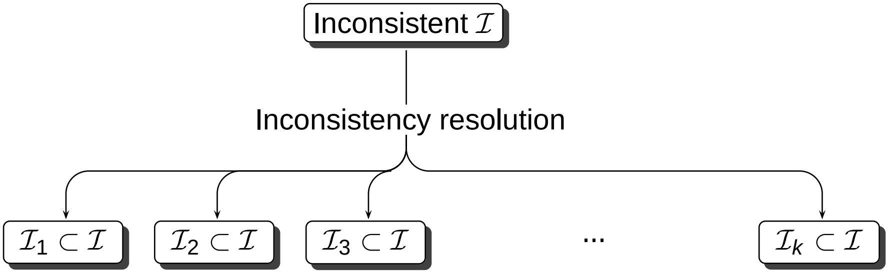
Disaggregation



Elicitation and Robustness



Inconsistency detection and resolution



Contents

Context of the problem

- Michel has just started a business `fun4all.com` to sell on the internet technological product (mp3, games consoles, ...) to young customers.
- He wants to optimize the product range to propose on his web site.
- He wants the products to be attractive for his young clients, but cannot afford to have a large catalog.
- To identify the mp3 to sell, Michel paid a marketing agency which evaluated (on the basis of a panel of young potential buyers) the perceived quality of mp3 players.

Context of the problem

- Each mp3 player was evaluated on three dimensions (storage capacity, autonomy, ergonomy/design) on a [0,100] scale on each dimension.
- A part of the results is synthesized hereafter.

	Dimension 1	Dimension 2	Dimension 3
a_1	10	50	70
a_2	34	56	84
a_3	40	90	45
a_4	30	10	70
a_5	60	80	45
a_6	49	56	54
⋮	⋮	⋮	⋮

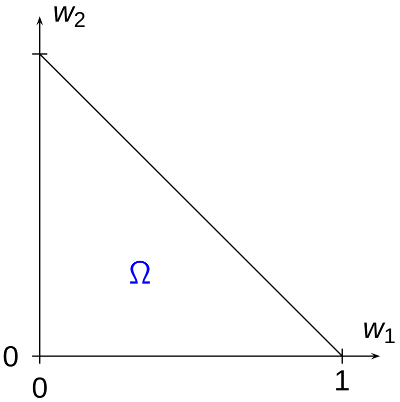
Context of the problem

- To appreciate the overall attractiveness of the mp3 players, Michel wants to define a ranking of the players grounded on a weighted sum.
- $g(a_i) = w_1.g_1(M_i) + w_2.g_2(M_i) + w_3.g_3(M_i)$ with $w_1 + w_2 + w_3 = 1$ and $w_i \geq 0, \ i = 1, 2, 3$.
- To elicit the preference model of his young customers, Michel will asks to his nephew Antonin some questions.

Contents

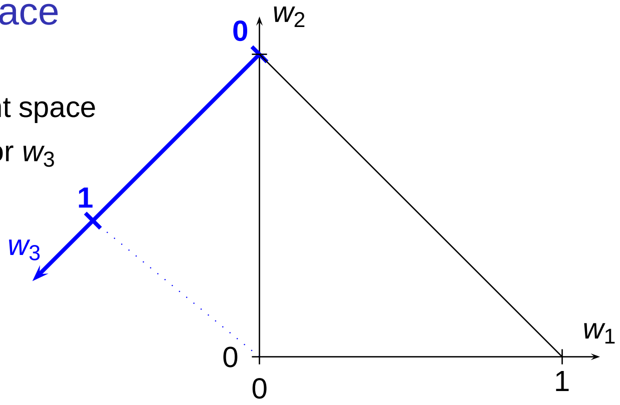
Weights space

- Weight space



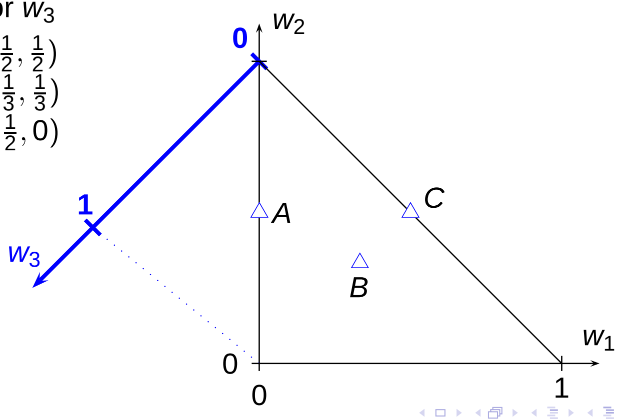
Weights space

- Weight space
- axis for w_3



Weights space

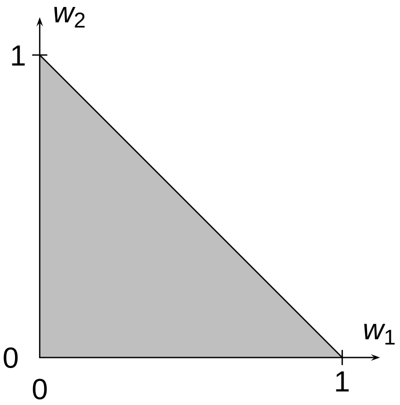
- Weight space
- axis for w_3
- A=$(0, \frac{1}{2}, \frac{1}{2})$
  B=$(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
  C=$(\frac{1}{2}, \frac{1}{2}, 0)$



Preference information

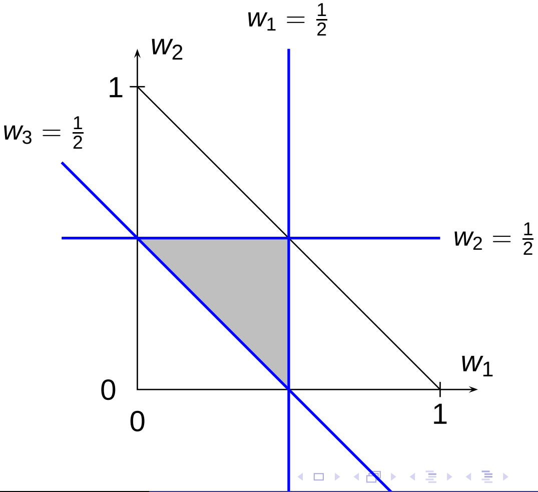
- Antonin has expressed the following preference statements:
  - The player a_1 is at least as attractive as a_4.
  - a_5 is not worse than a_3.
  - a_2 is at least as good as a_6.
- Moreover, Michel considers that each dimension should'nt represent more than half of the value of a player,

Induced weights space



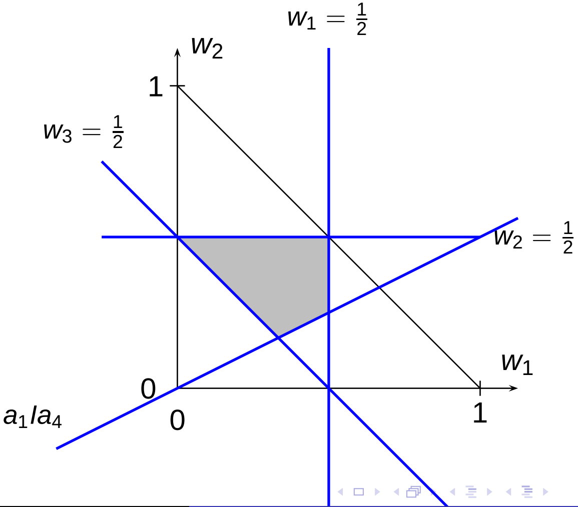
Induced weights space

- $w_j \leq \frac{1}{2}$



Induced weights space

- $w_j \leq \frac{1}{2}$
- $a_1 \succsim a_4$



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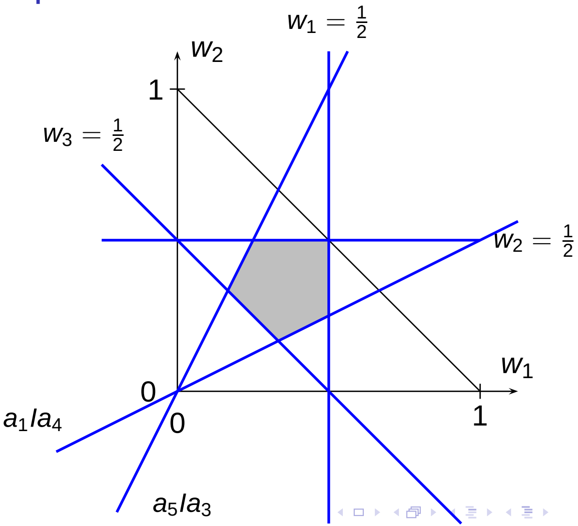

Induced weights space

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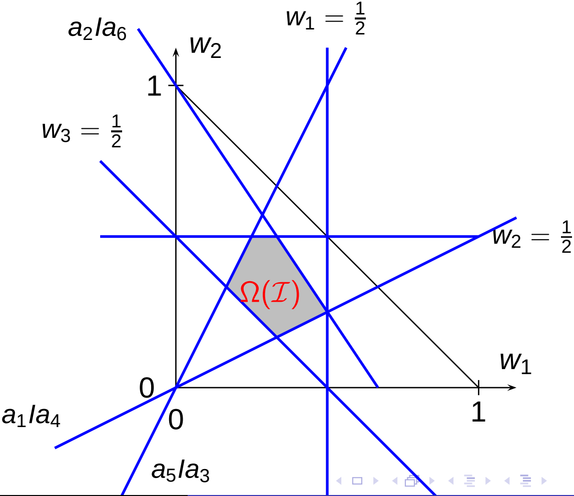
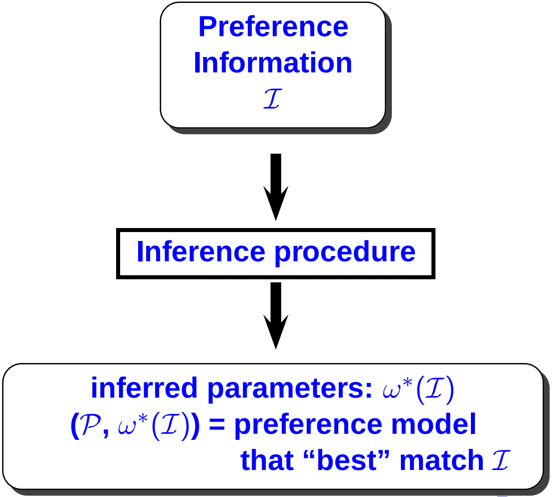
- $a_2 \succsim a_6$



Contents

Disaggregation



Disaggregation - Inference

- Choose $\omega^*(\mathcal{I}) \in \Omega(\mathcal{I})$, which maximize the minimum value among
  - $g(a_1) - g(a_4)$
  - $g(a_5) - g(a_3)$
  - $g(a_2) - g(a_6)$
  - $-w_i + 0.5,\ i = 1, 2, 3$

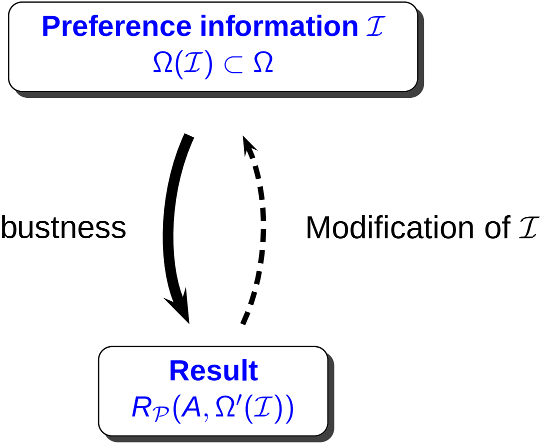
$$\begin{cases} Max & \alpha \\ \text{s.t.} & \alpha \leq g(a_1) - g(a_4) \\ & \alpha \leq g(a_5) - g(a_3) \\ & \alpha \leq g(a_2) - g(a_6) \\ & \alpha \leq -w_i + 0.5,\ i = 1, 2, 3 \\ & \omega \in \Omega(\mathcal{I}) \end{cases}$$

Disaggregation - Inference

- Illustration : `Fun4all-inference.xls`

Contents

Elicitation and Robustness



Elicitation and Robustness

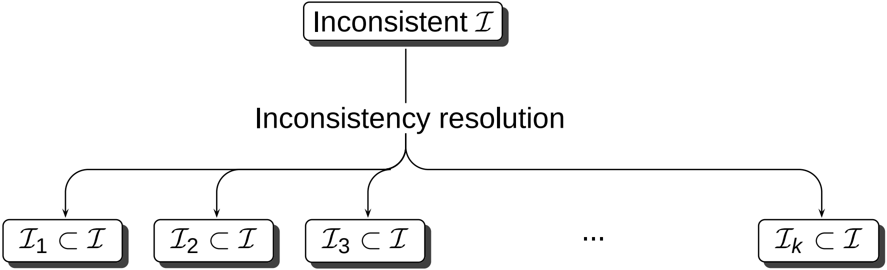
- Main difficulty : definition of a "*robust*" ranking,
- Suppose *a* is "robustly" ranked better than *b* if $g(a) > g(b), \; \forall \omega \in \Omega(\mathcal{I})$,
- To check whether *a* is "robustly" ranked better than *b*, we must maximize and minimize $g(a) - g(b)$ s.t. $\omega \in \Omega(\mathcal{I})$
- According to $Max_{\omega \in \Omega(\mathcal{I})}(g(a) - g(b)) > 0$ and $Min_{\omega \in \Omega(\mathcal{I})}(g(a) - g(b)) < 0$ the "robustly" ranking can be derived.

Robust results computation

- Illustration : `Fun4all-RobustRanking.xls`

Contents

Inconsistency detection and resolution

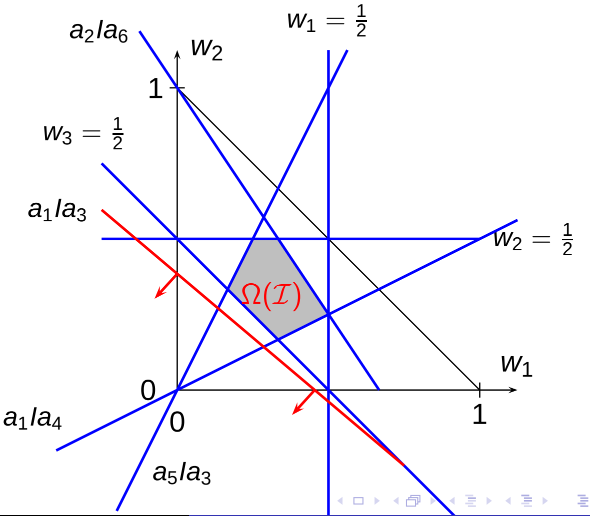


Inconsistency detection and resolution

- Suppose Antonin want to add the statement i = "a_1 is better than a_3",
- Observe that this new statement i contradicts the former preference information $\mathcal{I}$,
- Therefore $\Omega(\mathcal{I} \cup \{i\}) = \emptyset$,

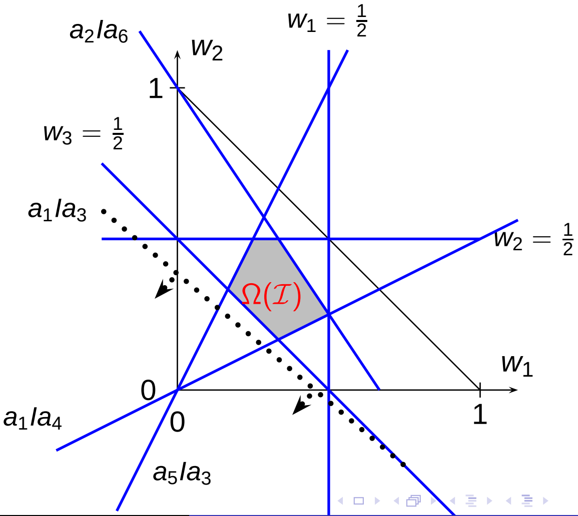
Empty weights space

- $w_j \leq \frac{1}{2}$
- $a_1 \succsim a_4$
- $a_5 \succsim a_3$
- $a_2 \succsim a_6$
- $a_1 \succsim a_3$



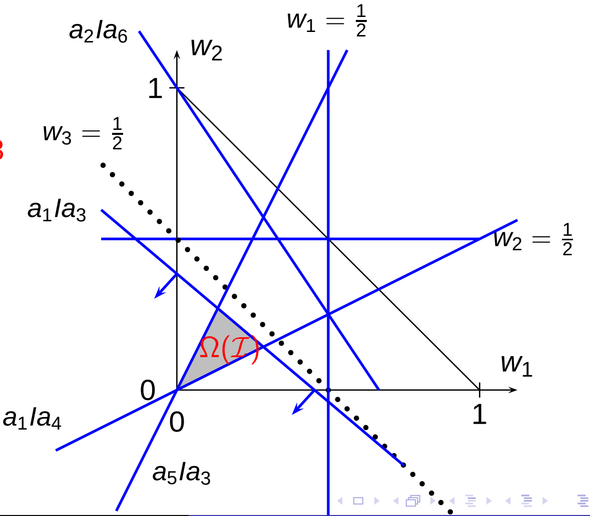
Inconsistency resolution: delete $a_1 \succsim a_3$

- $w_j \leq \frac{1}{2}$
- $a_1 \succsim a_4$
- $a_5 \succsim a_3$
- $a_2 \succsim a_6$
- ~~$a_1 \succsim a_3$~~



Inconsistency resolution: delete $w_1 \leq \frac{1}{3}$

- $w_j \leq \frac{1}{2}, j \neq 3$
- $a_1 \succsim a_4$
- $a_5 \succsim a_3$
- $a_2 \succsim a_6$
- $a_1 \succsim a_3$



Inconsistency resolution

- Inconsistency is detected when $Max_{w \in \Omega(\mathcal{I})} 0$ is infeasible.
- To solve the inconsistency, it is necessary to identify the maximal subsets of $\mathcal{I}$ yielding a non-empty polyhedron.
- In our example, consider the program (M is a large positive value):

$$\begin{array}{ll} Min & \sum y_i \\ \text{s.t.} & g(a_1) - g(a_4) + M.y_1 \geq 0 \\ & g(a_5) - g(a_3) + M.y_2 \geq 0 \\ & g(a_2) - g(a_6) + M.y_3 \geq 0 \\ & g(a_1) - g(a_3) + M.y_4 \geq 0 \\ & w_1 - M.y_5 \leq 0.5 \\ & w_2 - M.y_6 \leq 0.5 \\ & w_3 - M.y_7 \leq 0.5 \\ & y_i \in \{0, 1\} \end{array}$$

Inconsistency resolution

- Illustration : `Fun4all-Inconsistency.xls`