JPOC 14

14e Journées polyèdres et optimisation combinatoire
Caen (France)
10-11 juin 2025 : Cours doctoraux
12-13 juin 2025 : Conférences

OPEN PROBLEMS
(THAT I LIKE)

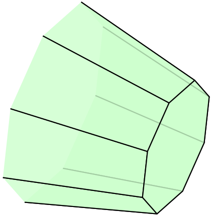
ROLAND GRAPPE

LAMSADE
UMR 7243

Dauphine | PSL
UNIVERSITÉ PARIS

Menu

POLYHEDRA, POLYTOPES, AND CONES



Polyhedron $=$ intersection of a finite number of half-spaces

$$= \{x \in \mathbb{R}^d : a_1^T x \geqslant b_1, \dots, a_m^T x \geqslant b_m\}$$

$$= \{Ax \geqslant b\}$$

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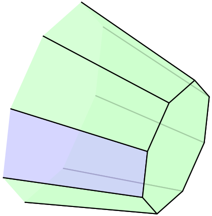

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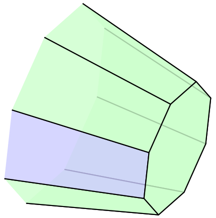
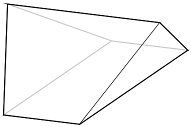
Facet = inequality of an irredundant description

Polyhedra, Polytopes, and Cones





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Polytope $=$ bounded polyhedron

$= \text{conv}(V)$

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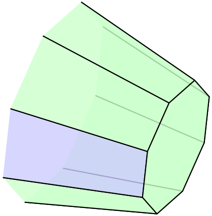

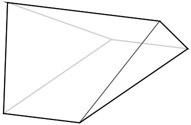

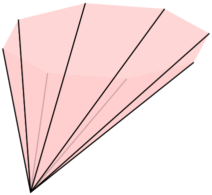

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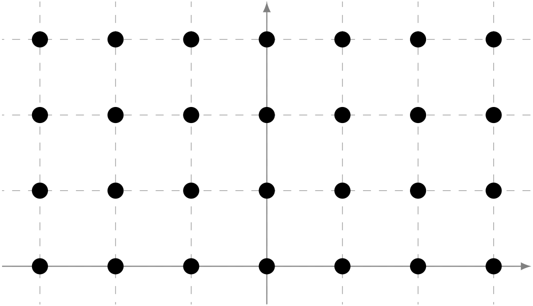
Cone $=$ polyhedron containing a point that lies on every facet

Integrality and Box-Integerality



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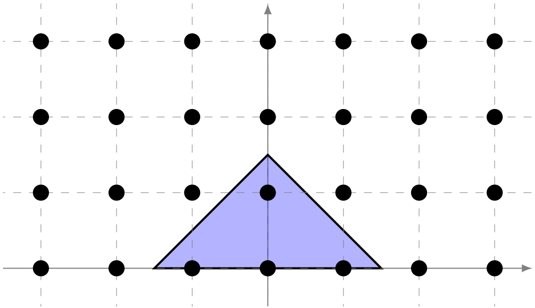

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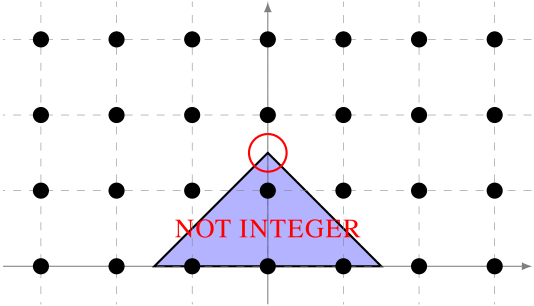

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- Integer: Each face of P contains an integer point

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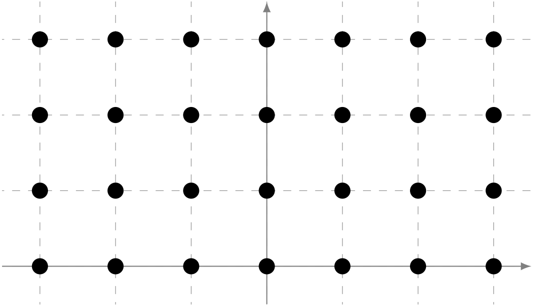

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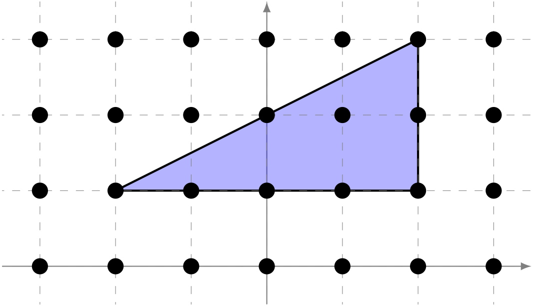

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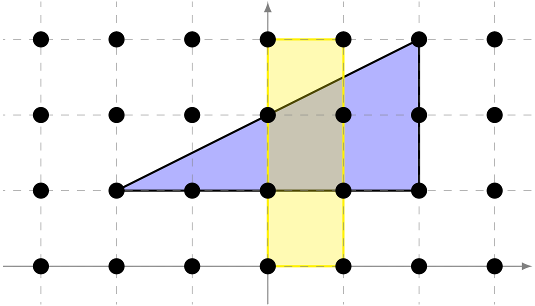

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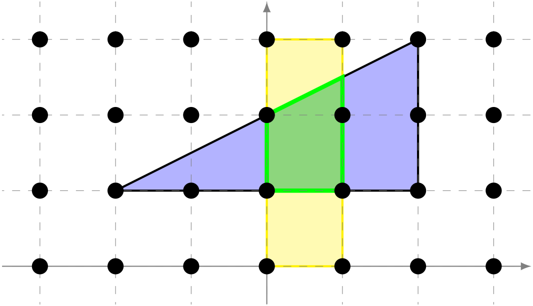

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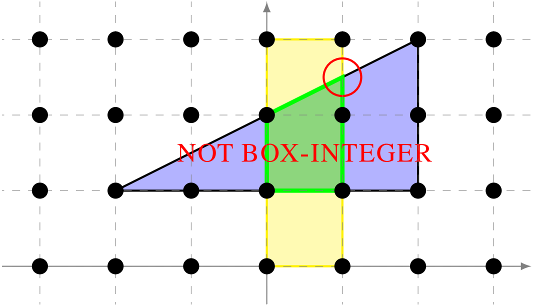

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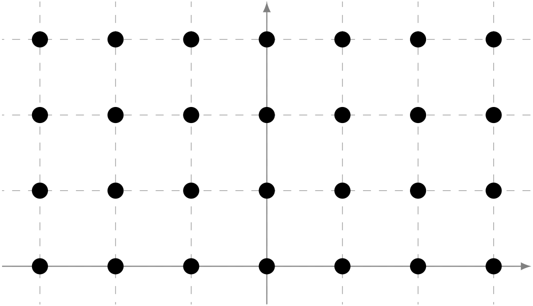

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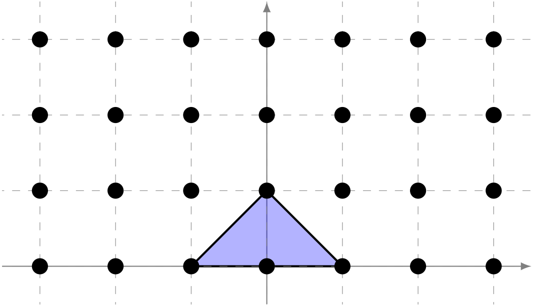

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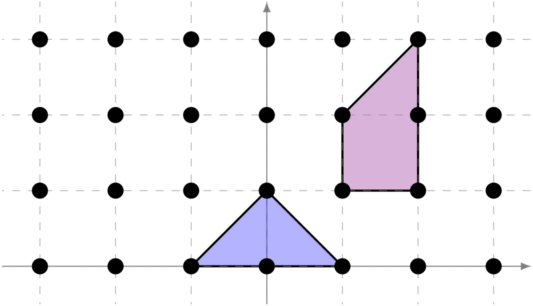

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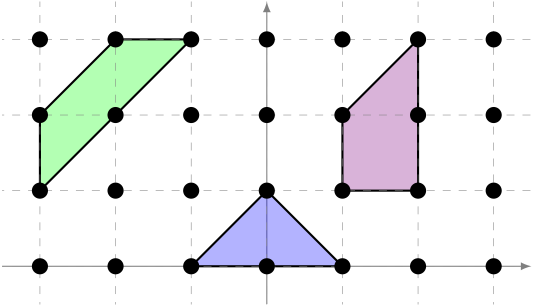

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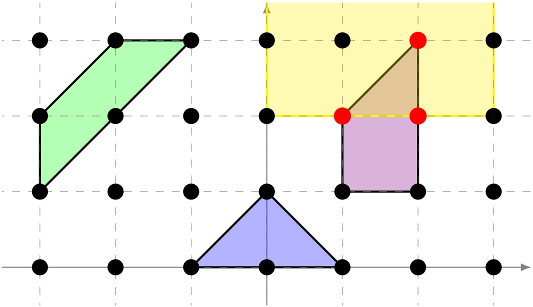

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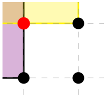
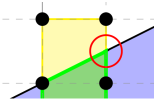
RECOGNIZING BOX-INTEGRALITY

OBS: A polyhedron is box-integer if and only if the points obtained by fixing integer coordinates in any face are all integer





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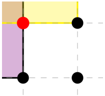

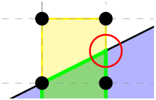

Deciding whether a polyhedron is box-integer is in co-NP

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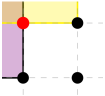

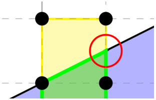

Deciding whether a polyhedron is box-integer is in co-NP

Obs: A polytope in the unit cube is integer if and only if it is box-integer

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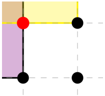

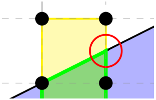

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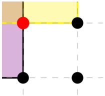

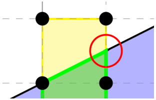

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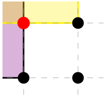

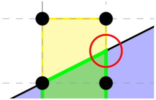

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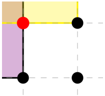

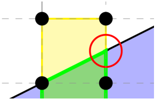

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11:20 - 11:50 Totally equimodular matrices: decomposition and triangulation. (Amphi S3 043) - M. Vallée

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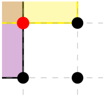

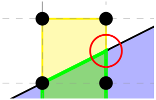

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- The simplicial case might be easier

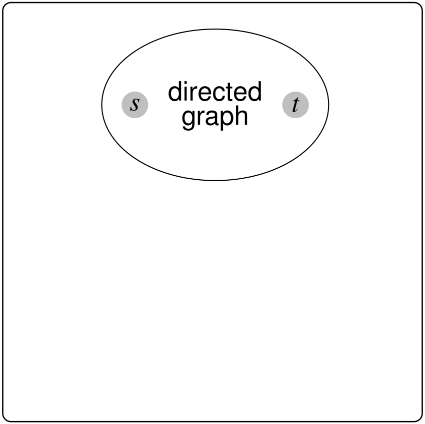
Menu

Box-Integrality

Box-Perfect Graphs

The r-Arborescence Polytope

Paths and Flows



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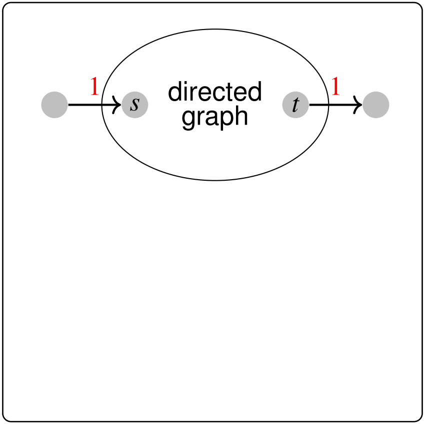

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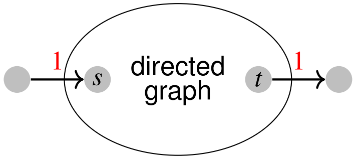
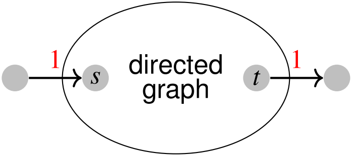


$\Downarrow$

arc-formulation

Paths and Flows



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integer polyhedron

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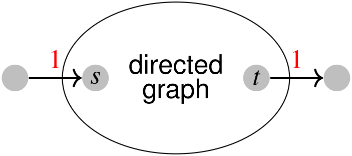

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st-path

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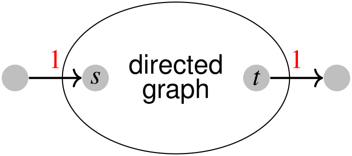

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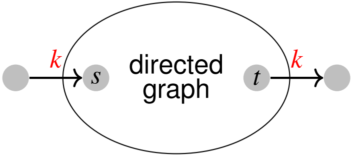

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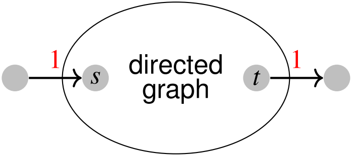

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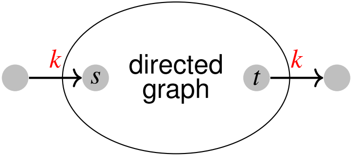

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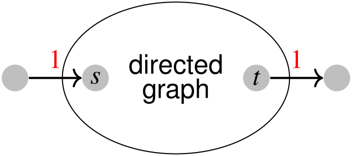

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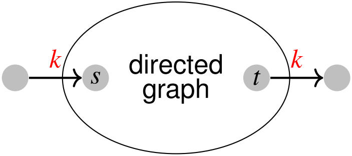

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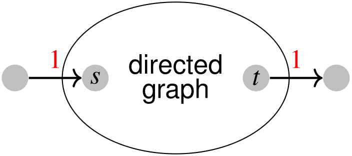

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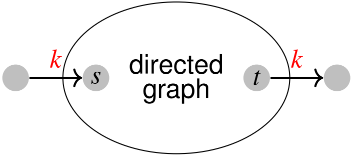

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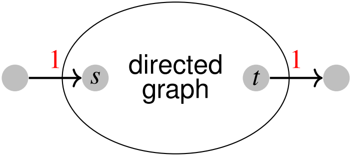
box-constraints (capacities)

PATHS AND FLOWS



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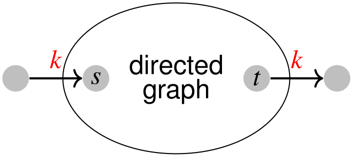

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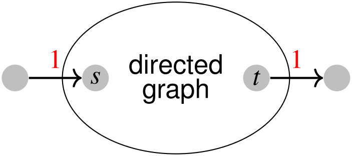
st-flow $=$ sum of st-paths

Paths and Flows



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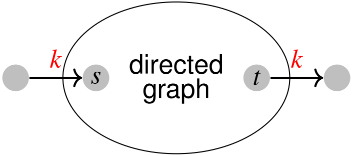

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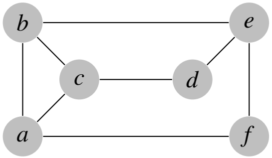
st-flow $=$ sum of st-paths

Theorem (Ford and Fulkerson – 1956)

The maximum st-flow equals the minimum capacity of an st-cut

Stable Sets, Cliques, and Perfect Graphs

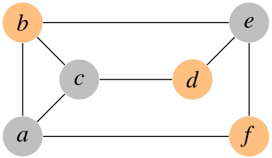
Stable sets



Stable set: set of pairwise nonadjacent vertices

Stable Sets, Cliques, and Perfect Graphs

Stable sets



Stable set: set of pairwise nonadjacent vertices

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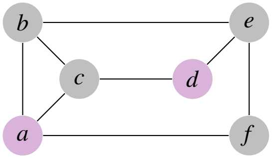

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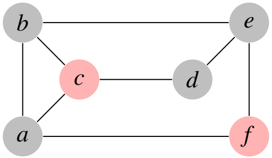

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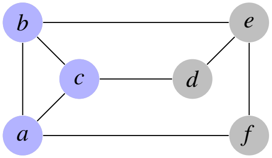

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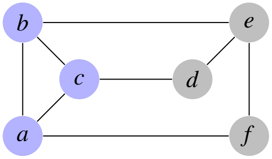

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Obs: $|\text{clique} \cap \text{stable}| \leqslant 1$

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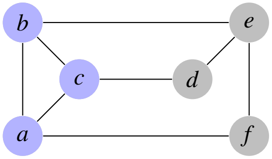

$\Downarrow$

clique-formulation

$$\begin{aligned} x(C) &\leqslant 1 \quad \forall C \text{ clique} \\ x &\geqslant 0 \end{aligned}$$

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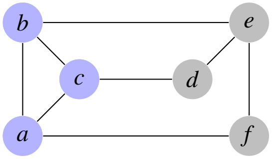

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max clique $=$ min coloring

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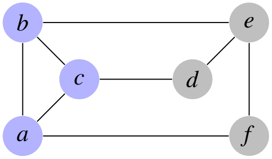

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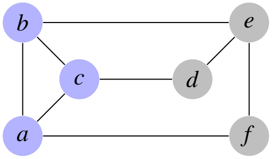
A graph is perfect if and only if the clique-formulation is its stable set polytope

STABLE SETS, CLIQUES, AND PERFECT GRAPHS

Stable sets



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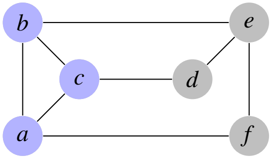

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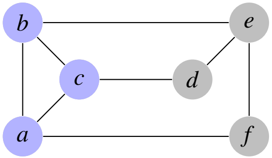

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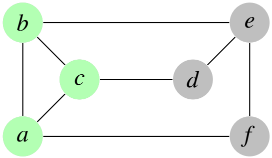

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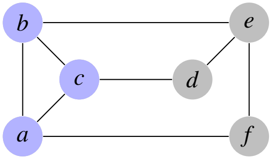

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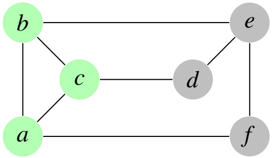

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$$\begin{aligned} x(C) &\leqslant k \quad \forall C \text{ clique} \\ x &\geqslant 0 \end{aligned}$$

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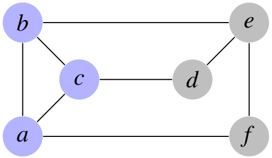

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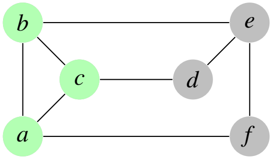

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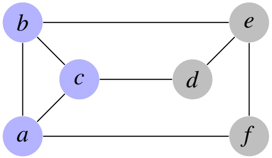

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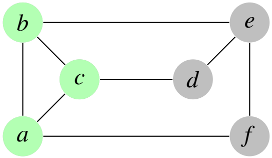

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box-integer polyhedron for
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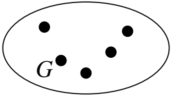
Box-perfect graphs in handwaving

The graphs in which stable sets satisfy a kind of "MaxFlow-MinCut" theorem

Box-Perfect Graphs

Open Problem (Cameron and Edmonds – 1982)

Characterize box-perfect graphs by forbidding induced subgraphs



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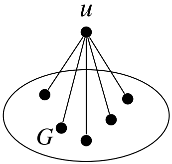

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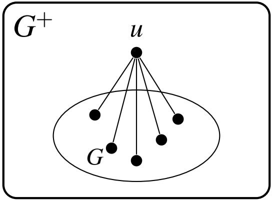

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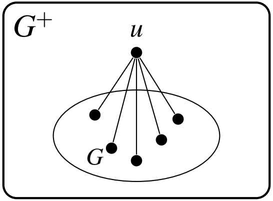

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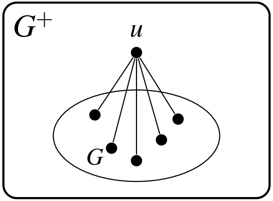

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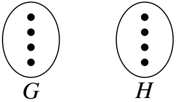

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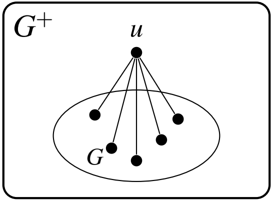

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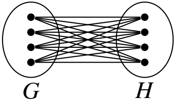

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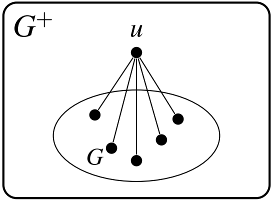

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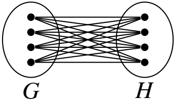

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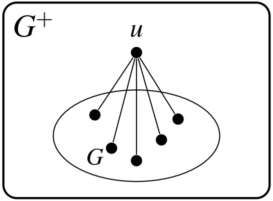

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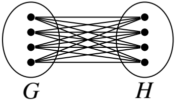

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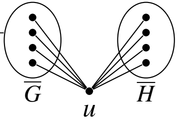

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Menu

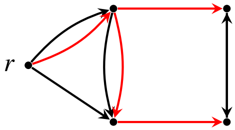
Box-Integrality

Box-Perfect Graphs

The r-Arborescence Polytope

ARBORESCENCES

IN A DIRECTED GRAPH $D = (V, A)$ WITH A ROOT $r \in V$

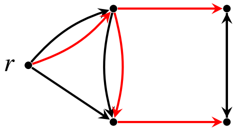


An r-arborescence is a set of arcs $B \subseteq A$:

- whose underlying undirected graph is a spanning tree
- in which every vertex except r receives exactly one arc

Arborescences

In a directed graph $D = (V, A)$ with a root $r \in V$



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Theorem (Edmonds – 1967)

The r-arborescence polytope is:

$$ARB = \left\{ x \in \mathbb{R}^A : \begin{array}{rcll} x(A) & = & |V| - 1 & \\ x(\delta^-(U)) & \geqslant & 1 & \text{for all } U \not\ni r \\ x & \geqslant & 0 & \end{array} \right\}$$

Sums of Arborescences

Theorem (Edmonds – 1967)

Every integer B in

$$kARB = \left\{ x \in \mathbb{R}^A : \begin{array}{rcl} x(A) & = & k(|V|-1) \\ x(\delta^-(U)) & \geqslant & k \quad \text{for all } U \not\ni r \\ x & \geqslant & 0 \end{array} \right\}$$

is the sum of k arborescences

$$B = \underbrace{B_1 + \cdots + B_k}_{r\text{-arborecences}}$$

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Open Problem (Sebő 1991, Gijswijt and Regts 2012)

Can every integer B in $kARB$ be written as:

$$B = \underbrace{\underbrace{B_1 + \cdots + B_1}_{t_1 \text{ copies}} + \cdots + \underbrace{B_{|A|} + \cdots + B_{|A|}}_{t_{|A|} \text{ copies}}}_{B_1, \ldots, B_{|A|}:\ r\text{-arborescences, and } t_1 + \cdots + t_{|A|} = k} \ ?$$

Recent Related Results

The spannning tree polytope is a matroid polytope

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The spannning tree polytope is a matroid polytope

Matroid polytopes

- Have the ICP (Gijswijt and Regts 2012)
- Have a regular unimodular Hilbert triangulation (Backman and Liu 2024)

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The r-arborescence polytope is the intersection of two matroid polytopes

Open Problem (Gijswijt and Regts 2012)

Does the intersection of two matroid polytopes have the ICP?

Meta-Question

Pick your favorite combinatorial object:
When does it satisfy some kind of "MaxFlow-MinCut" theorem?

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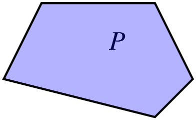

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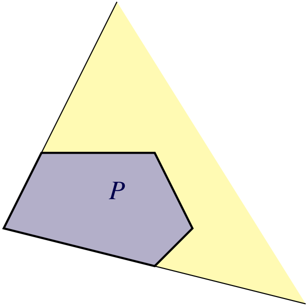

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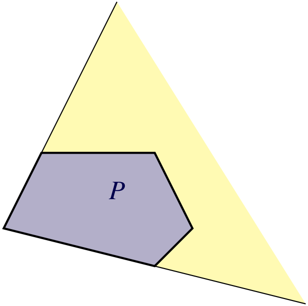

Minimal cones of the r-arborescence polytope
and of the stable set polytope of a box-perfect graph are box-integer

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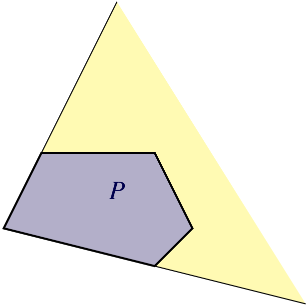

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Thank you for your attention!