Extension of vertex cover and independent set in some classes of graphs and generalizations

Mehdi Khosravian

Joint work with: Katrin Casel, Henning Fernau, Jérôme Monnot and Florian Sikora

May 27, 2019

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Solution: Some (inclusion-wise) minimal vertex cover S of G such that $S \supseteq U$,

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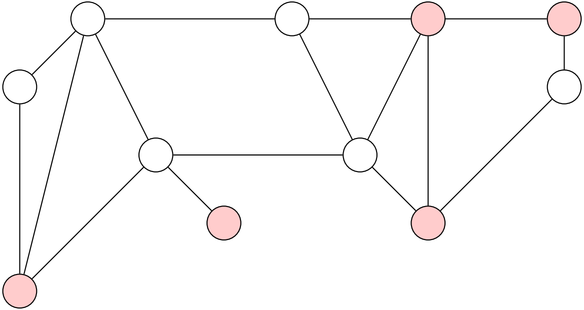

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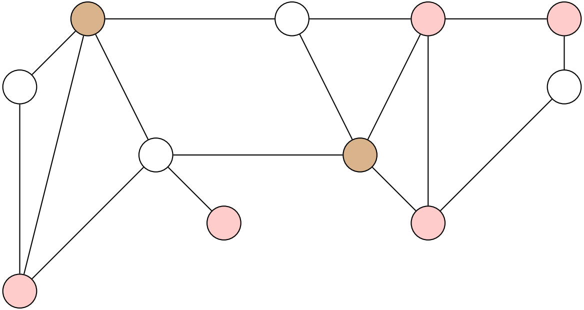

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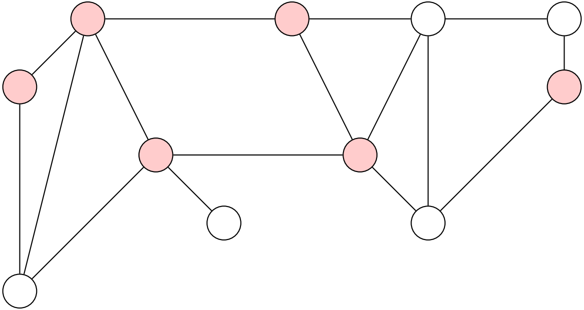

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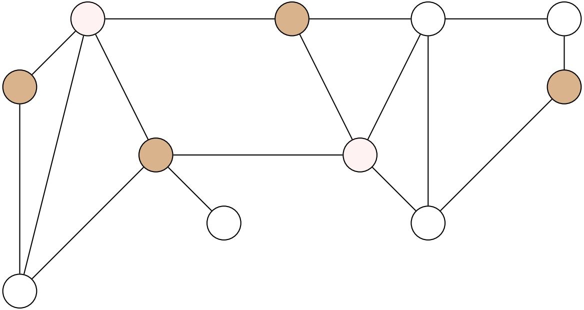

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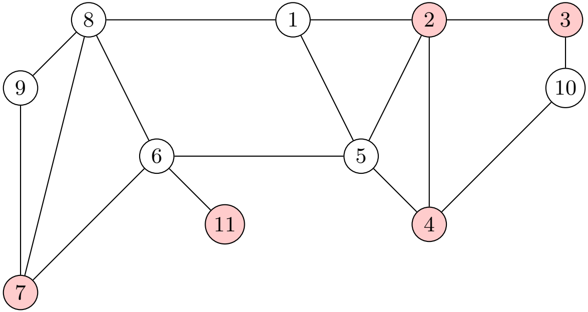

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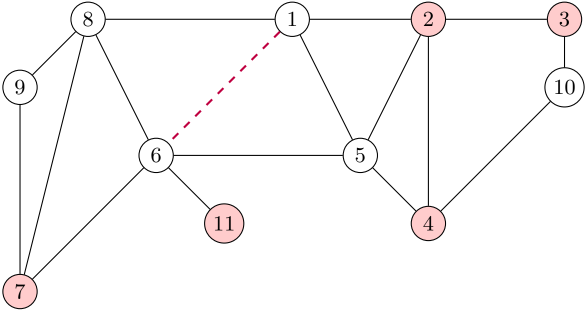

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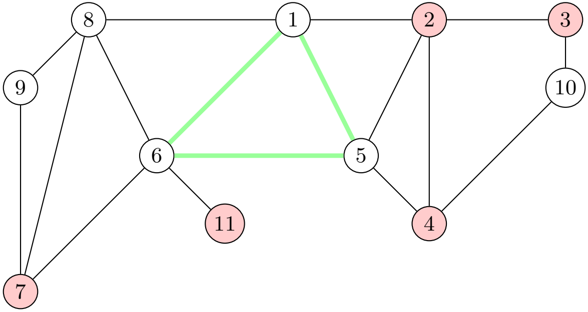

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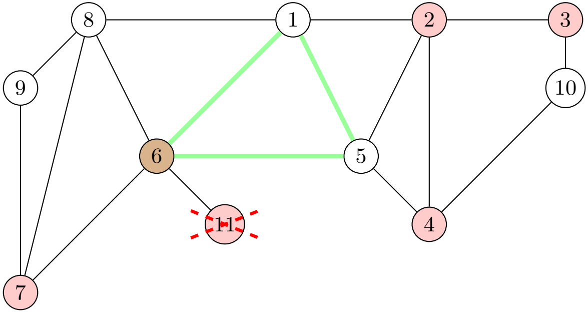

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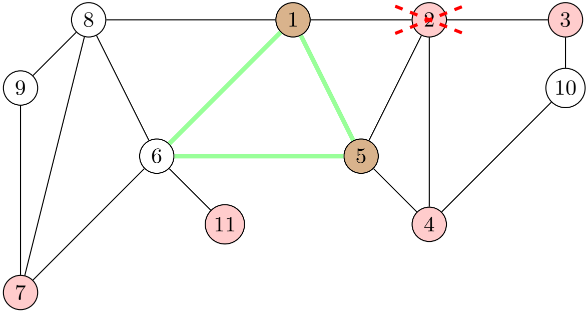

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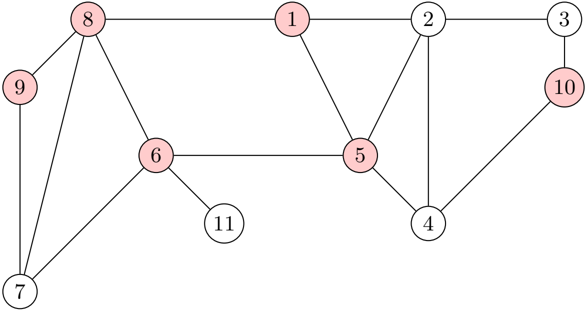

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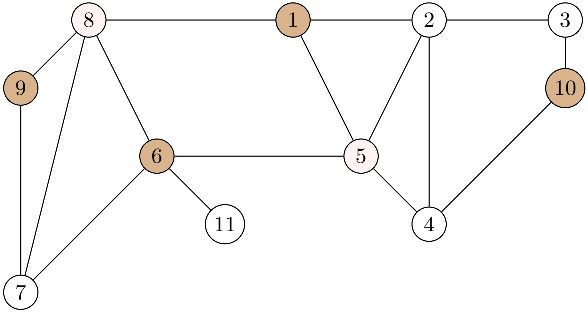

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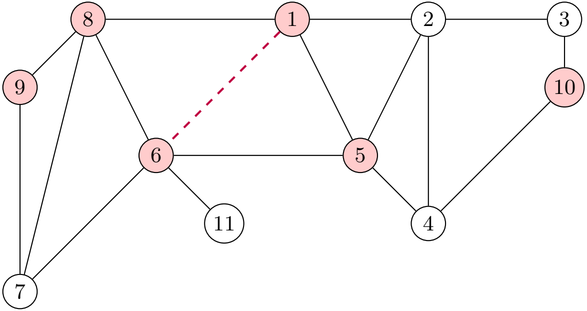

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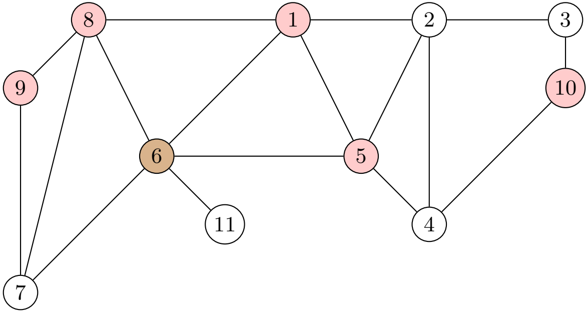

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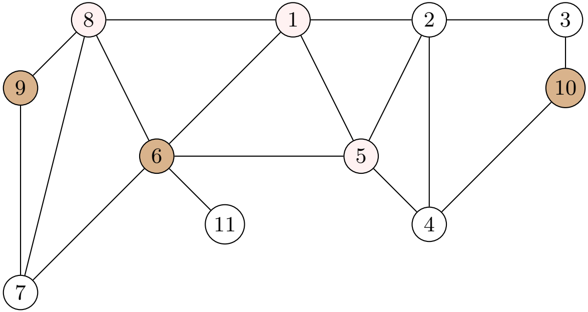

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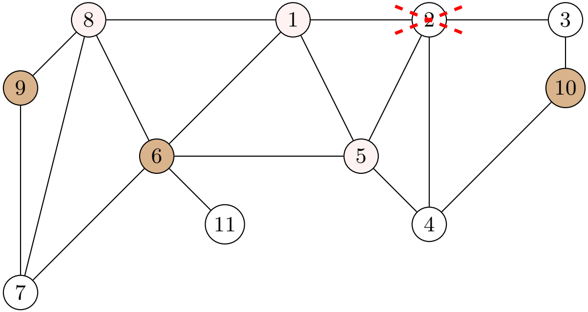

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Given as input $X, Y \subseteq V$, is there any maximum independent set that includes X and does not intersect with Y.

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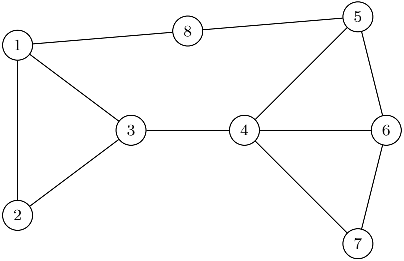

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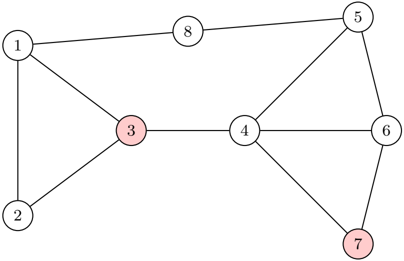

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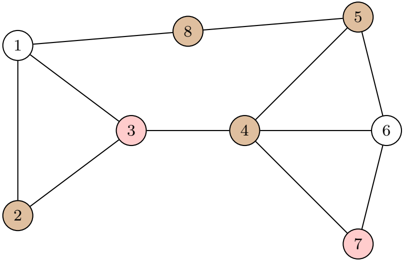

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- Start from an instance $I = (\mathcal{C}, \mathcal{X})$ from $(3, B2)$-SAT
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2-Balanced 3-SAT (or $(3, B2)$-SAT)

• each clause has exactly 3 literals,
• each variable appears exactly 4 times, twice negative and twice positive.

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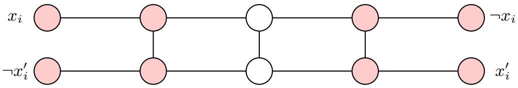

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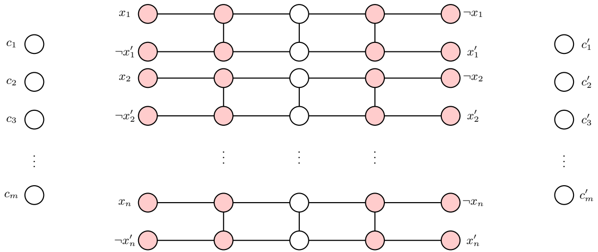

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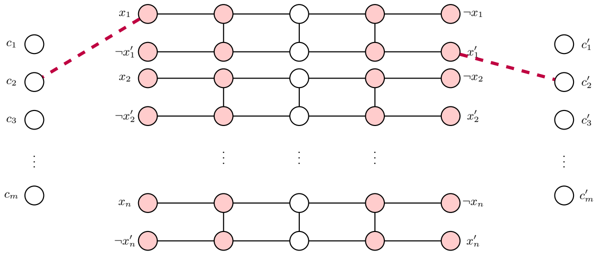

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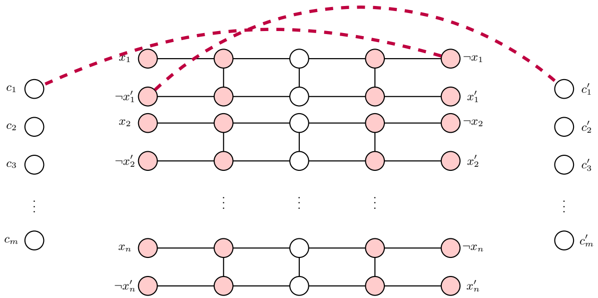

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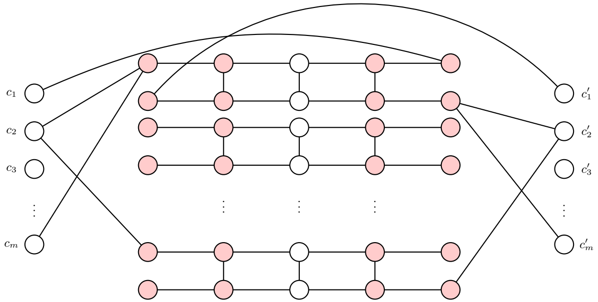

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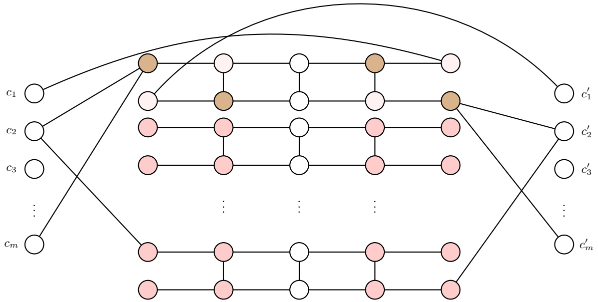

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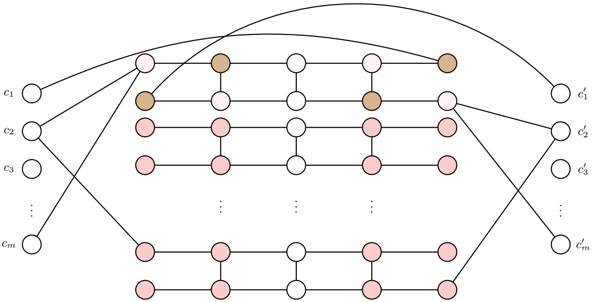

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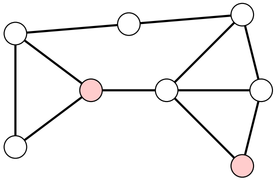

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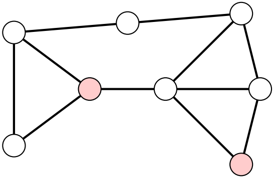

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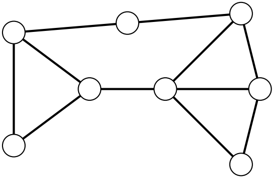

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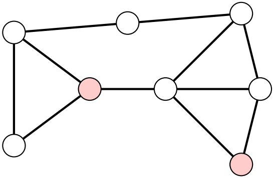

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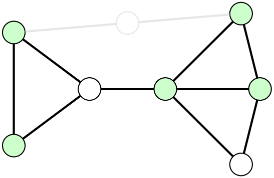

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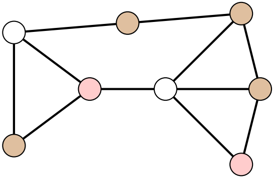

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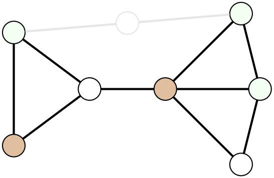

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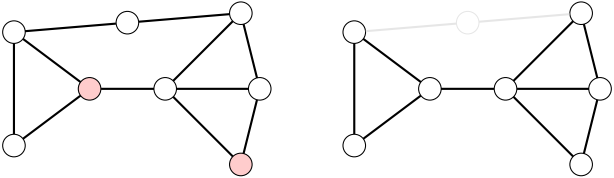

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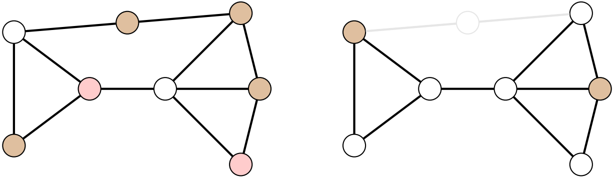

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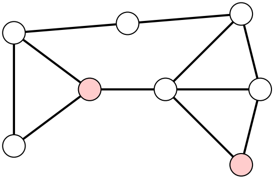

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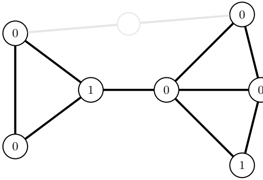

Parameterized Complexity: Planar and Bipartite

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EXT VC parameterized by $|U|$ is W[1]-complete in bipartite graphs.

Price of Extension

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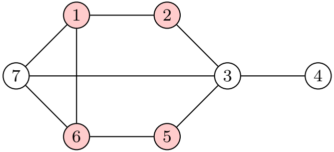
EXT VC



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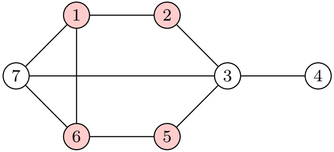

$\text{EXT}_{\text{MAX}}\text{VC}$

Input: $G = (V, E)$ and $U \subseteq V$.

Solution: Minimal vertex cover S of G.

Goal: Sol S that maximizes $|S \cap U|$.

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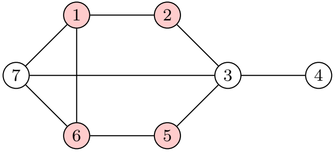

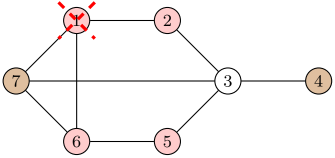

$|S \cap U| = 3$

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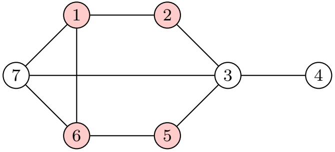

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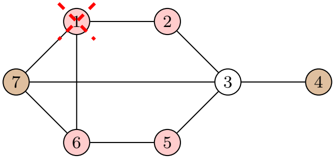

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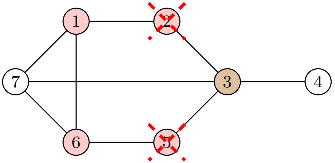
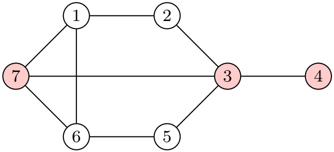


$|S \cap U| = 2$

Price of Extension

EXT IS



Price of Extension

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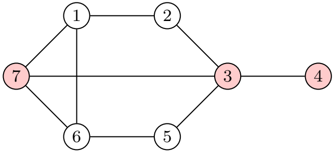

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Price of Extension

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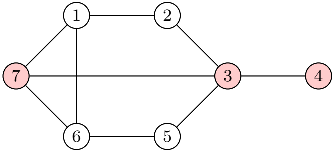

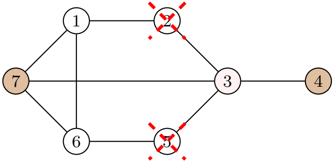

$|S \cup U| = 5$

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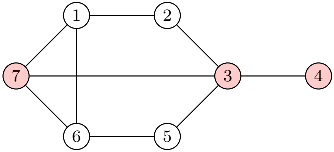

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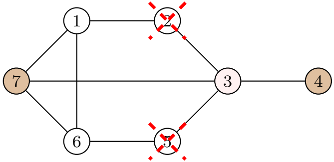

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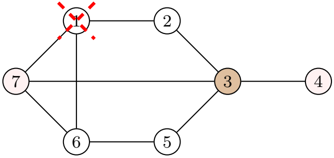

$|S \cup U| = 4$

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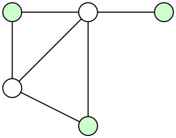

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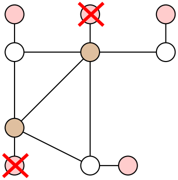

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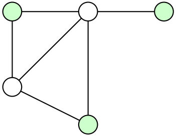

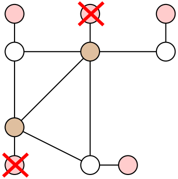

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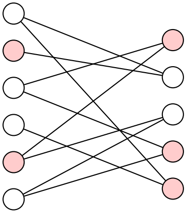

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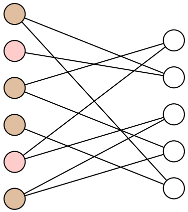

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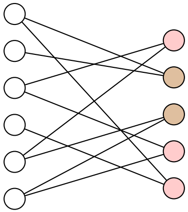

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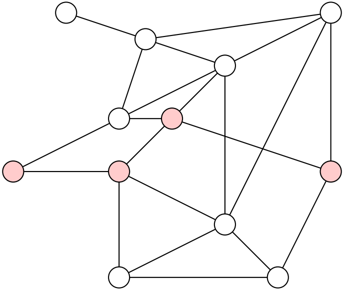

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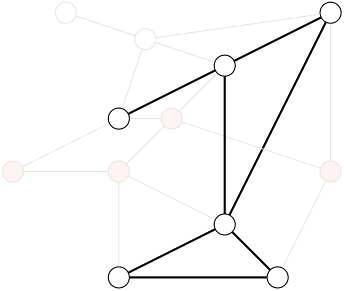

maximum degree $\leq \Delta - 1$

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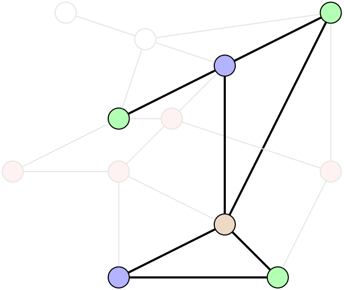

colored by $\leq \Delta$ colors

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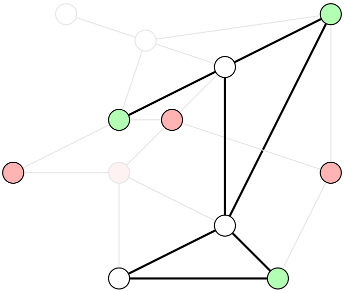

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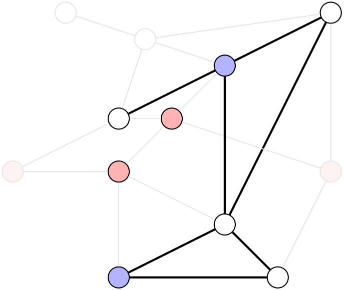

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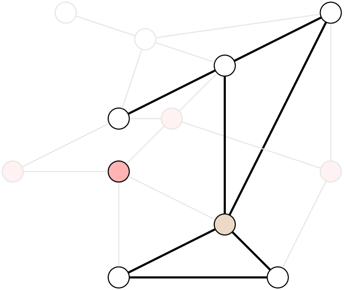

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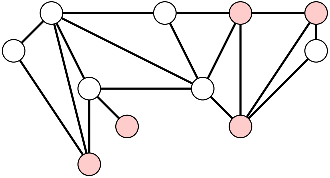
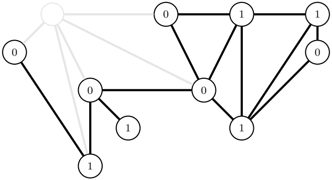


EXT$_{\text{MAX}}$VC



ISDS

$$OPT_{\mathbf{EXT_{max}VC}}(G, U) = |U| - OPT_{ISDS}(G')$$

Price of Extension: Positive Results

- Bipartite graphs $\rightarrow$ $\frac{1}{2}$-apx
- Bounded degree Δ $\rightarrow$ $\frac{1}{\Delta}$-apx
- Chordal graphs $\rightarrow$ polynomial

Conclusion

- EXT VC and EXT IS are NP-complete in bipartite, planar,... while are polynomial decidable in chordal, circular arc.
- EXT VC and EXT IS are W[1]-complete in bipartite graphs and are in FPT in planar graphs parameterized by $|U|$ and $|V \setminus U|$ respectively.
- $\text{EXT}_{\text{MAX}}\text{VC}$ and $\text{EXT}_{\text{MIN}}\text{IS}$ are equivalent and difficult to approximate in general, but easier on special graph classes (most notably, exactly solvable in polynomial time on chordal graphs).

Thanks for your attention!