



Methods and Models for Decision Making

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God in 7 steps:

- MCDM: a logical path
- Definition of the alternatives
- Choice of the attributes
- Determination of the utility functions
- Preference structure and weights
- Ranking + sensitivity analysis

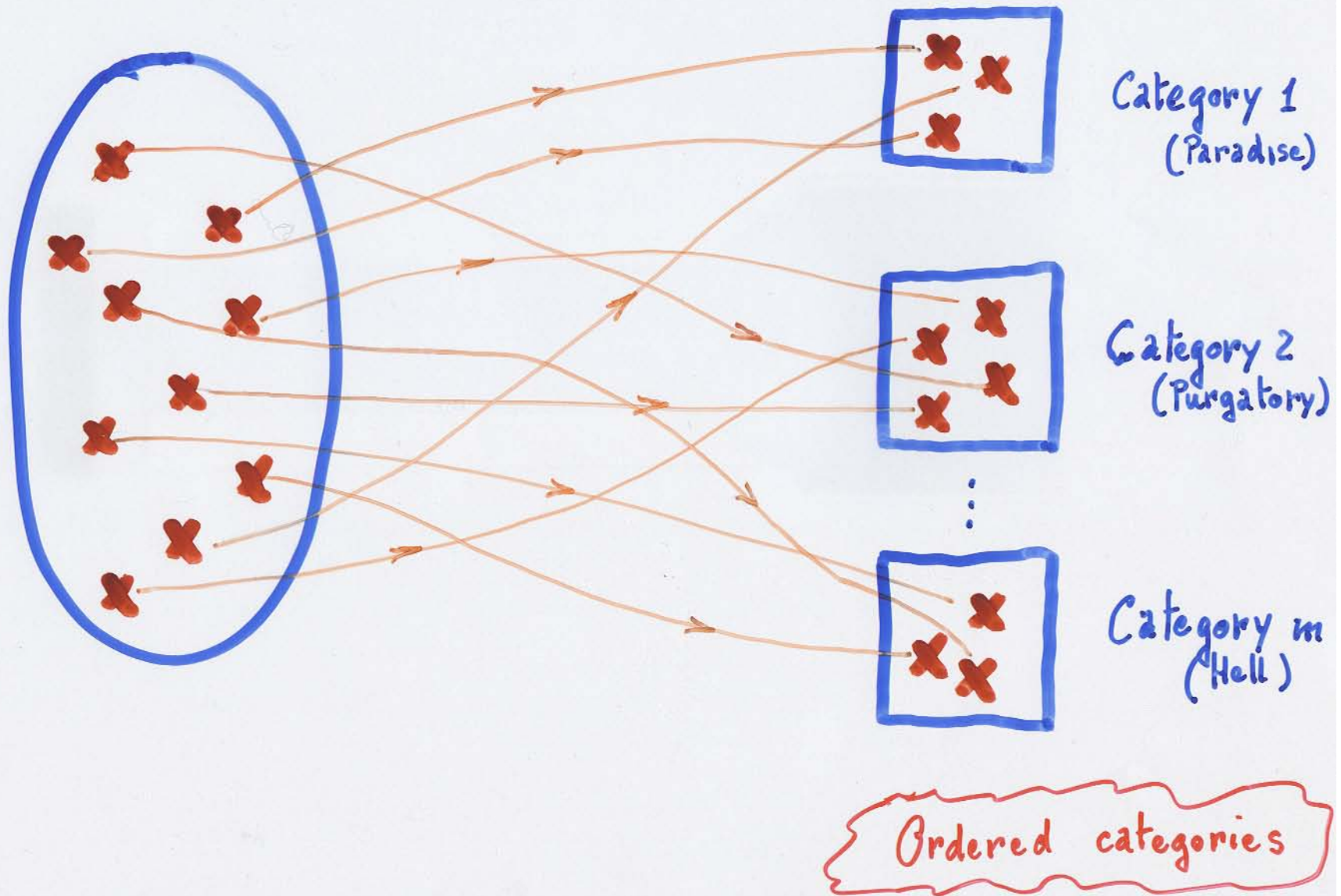
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- (5) Classification
- (6) Ranking-1, risk analysis
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- (8) A tentative case
- **(9) Rating problems**
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- *(11) Group decision*
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Rating problems

1. Rating (sorting)
2. An example
3. Definition of...
4. Comparison between objects and profiles
5. When $K \leq P_{ij}$
6. Threshold α (and winning coalitions)
7. Students (A, B, C, D)
8. Rating revised
9. About the method
10. More concerning winning coalitions
11. Test and conclusions

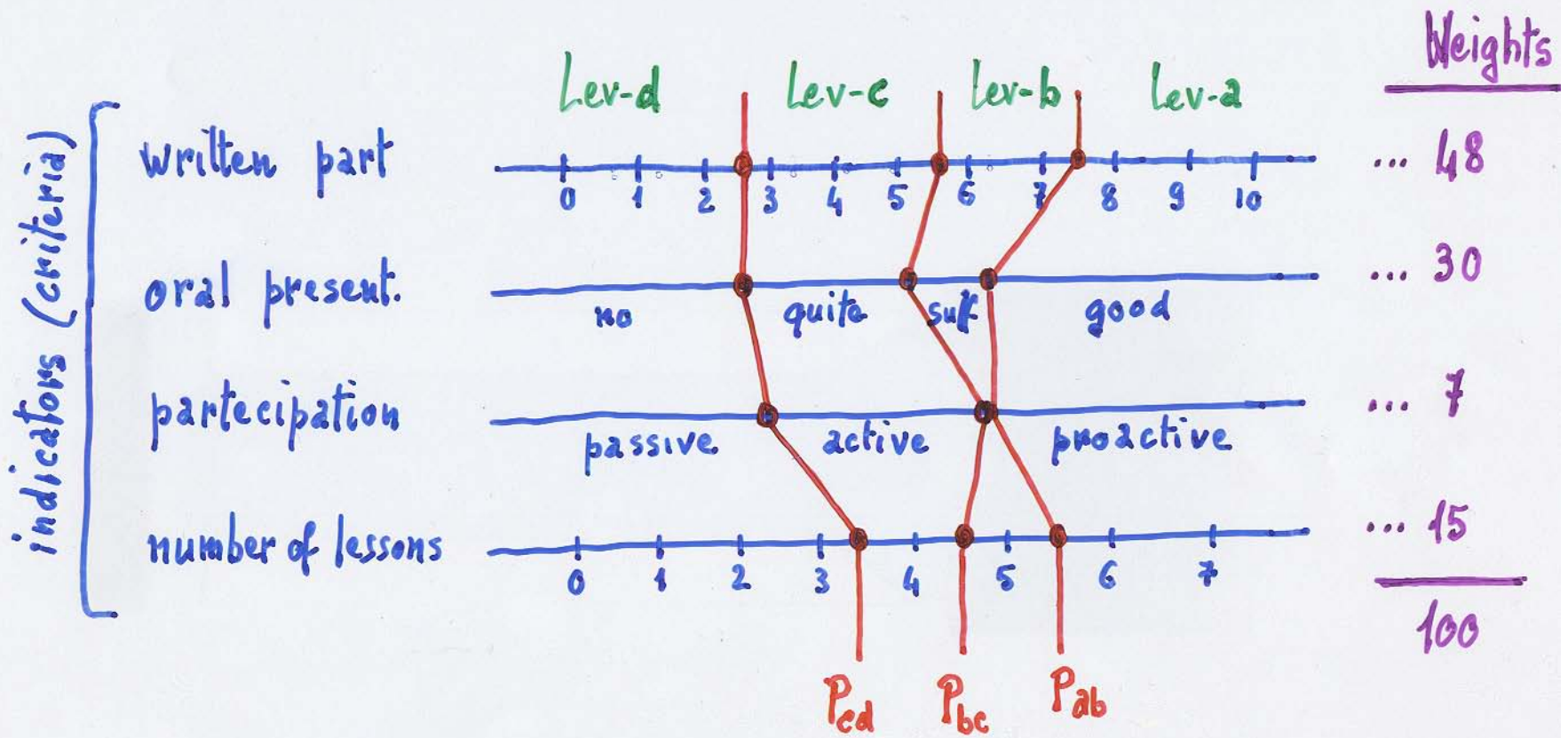
Rating (sorting)



An example

- Objects to be rated → the PhD students of MMDM
- Categories (levels)
 - Lev-a = excellent
 - Lev-b = good
 - Lev-c = sufficient
 - Lev-d = insufficient
- How implement a procedure? → the logical (& subjective) steps

Definition of...



... of

- indicators (criterià)
- weights
- levels (categories)
- profiles (= levels - 1)

profile = watershed between 2 categories

Comparison between objects and profiles

Student K
vs
profile P_{ij}

S outrank.
 \sim indiffer.
? incompar.



$K \ S \ P_{ij}$



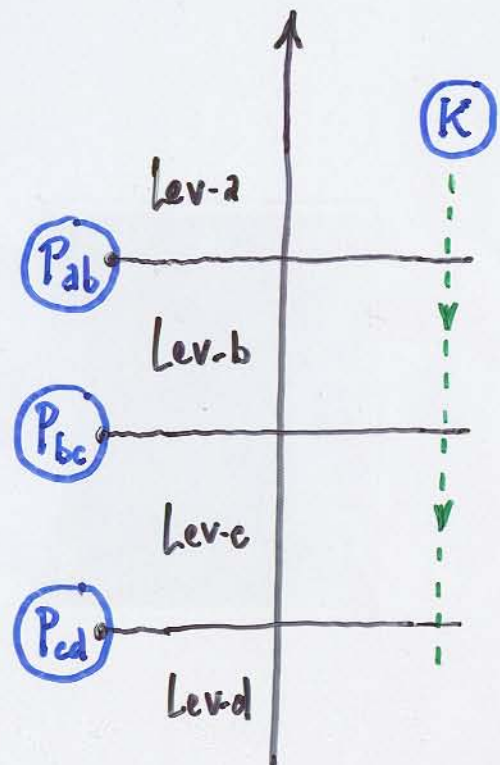
$P_{ij} \ S \ K$



$K \ \sim \ P_{ij}$



$K \ ? \ P_{ij}$



Procedure: K is
in Lev- i iff

$$\begin{cases} K \ S \ P_{ij} \text{ (or)} \\ K \ \sim \ P_{ij} \end{cases}$$

- i. Reasons in favor (concordance) \rightarrow HIGH ($\geq \alpha$)
- ii. Reasons against (discordance) \rightarrow LOW ($\leq \beta$)
- iii. Strong opposition (veto) \rightarrow NOT PRESENT



(in our case)

- i. Σ weights in favor of $K \geq \alpha$ (threshold to be fixed)
- ii. --
- iii. If number of lessons $< 4 \rightarrow$ veto $K \geq P_{cd}$ (so K in Lev-d)

Threshold α (and winning coalitions)

C1	_____	48
C2	_____	30
C3	_____	7
C4	_____	15

(Weights)

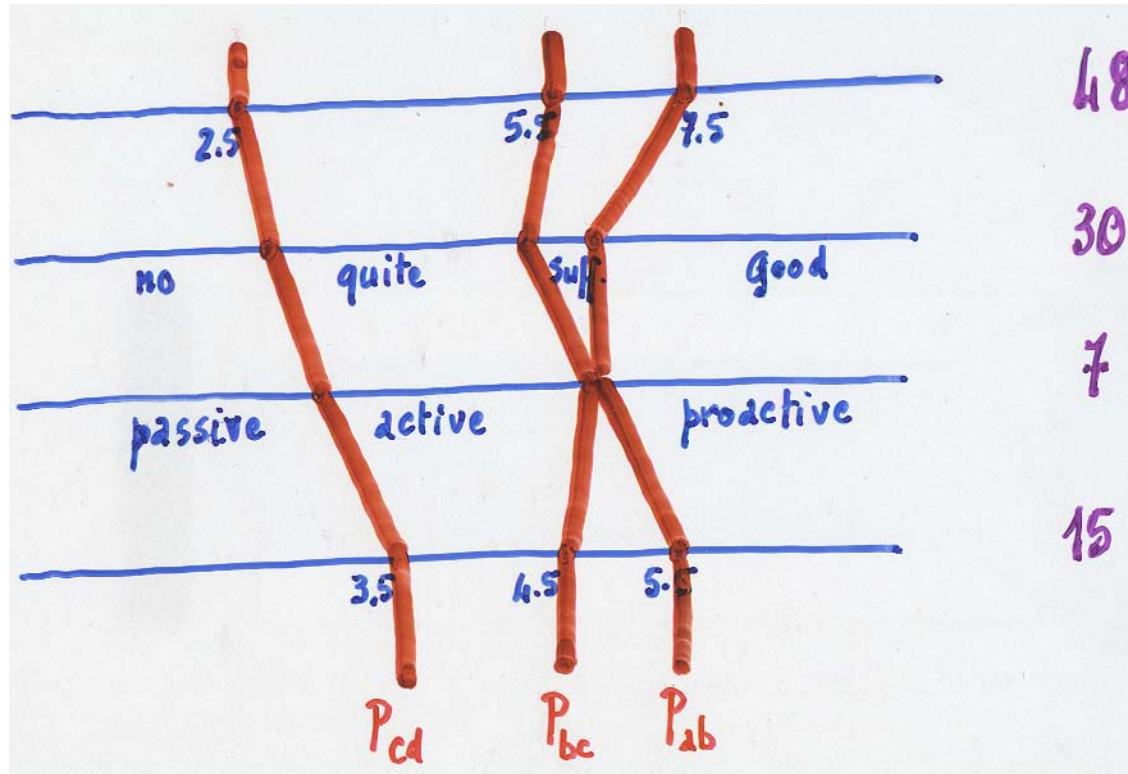
Let we fix
 $\alpha = 70$

- C1 + C2 + ...
- C1 + C3 + C4

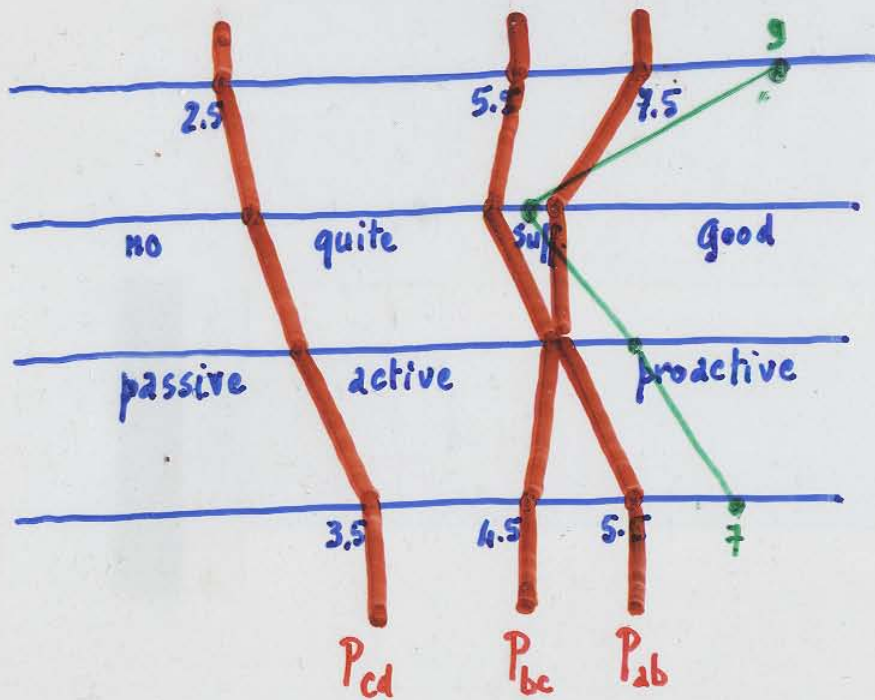
Without c2 no coalition
with $\sum w_i \geq \alpha$
(winning coalition)

c1	c2	c3	c4	$\sum w_i$
1	0	0	0	48
1	0	0	1	63
1	0	1	0	55
1	0	1	1	70 •
1	1	0	0	78 •
1	1	0	1	93 •
1	1	1	0	85 •
1	1	1	1	100





Student A

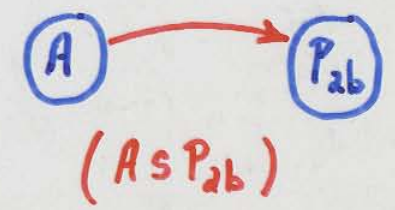


48
30
7
15

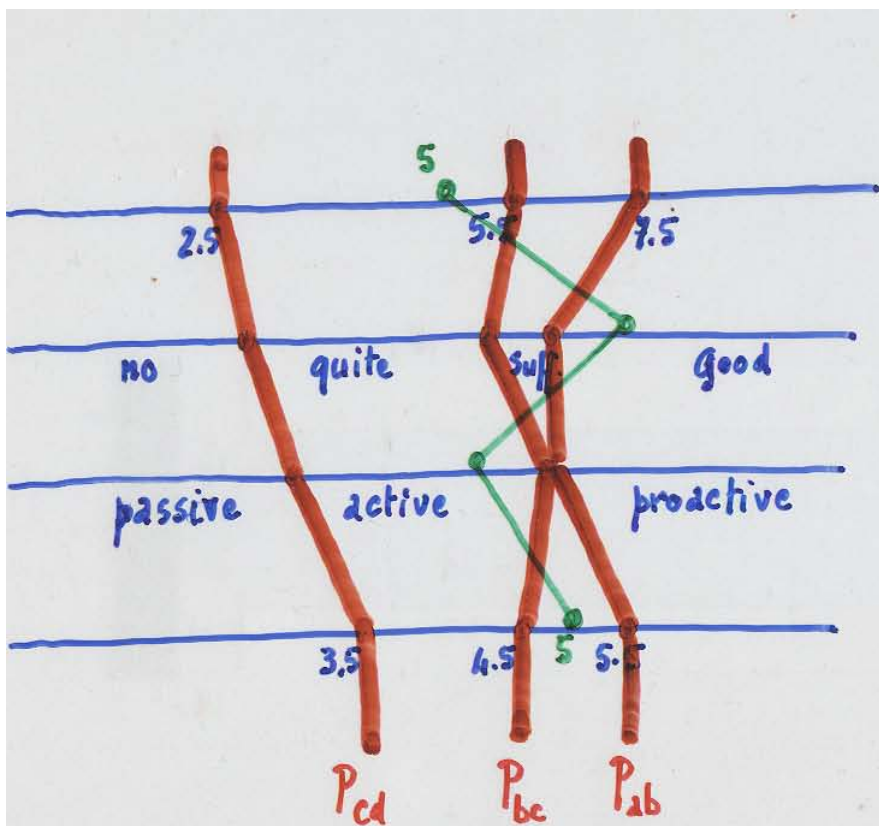
A ∈ Lev-a (excellent)



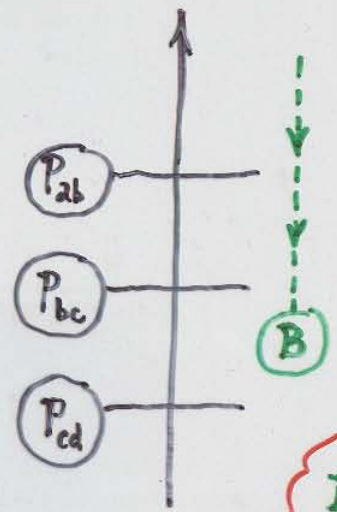
A vs. P_{ab} → concordance 70
veto → No



Student B



48
30
7
15

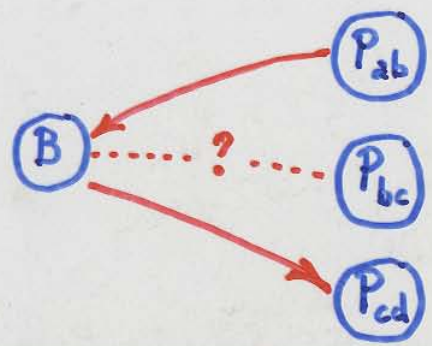


B ∈ Lev-c (suff.)

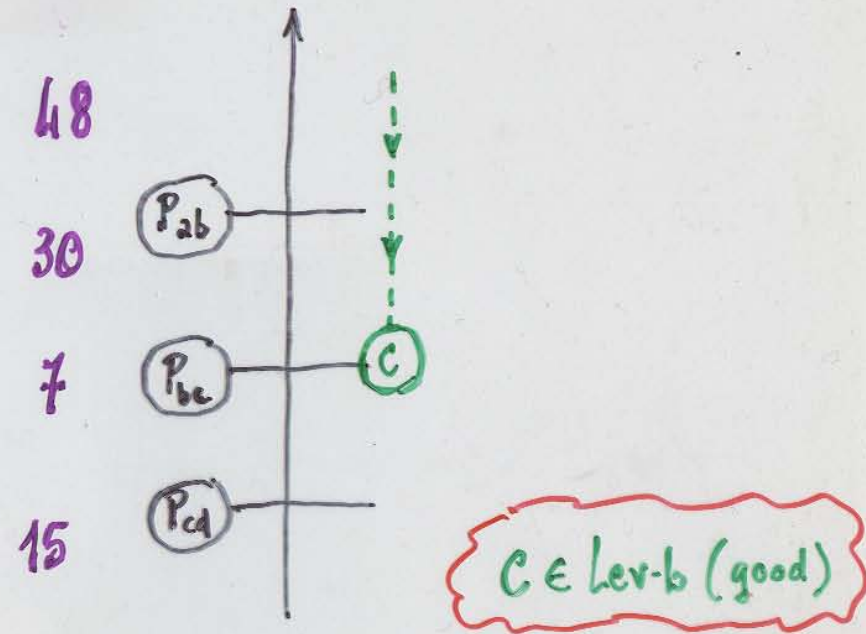
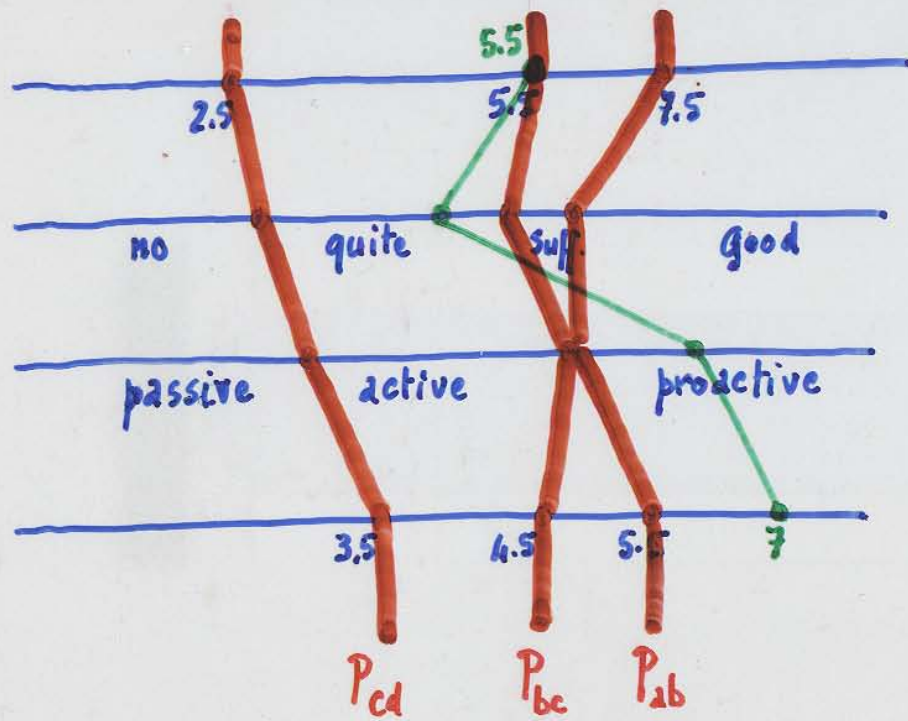


B vs

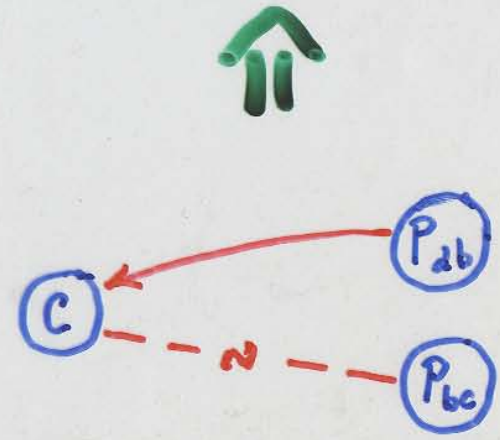
- $P_{ab} \rightsquigarrow P_{ab} \leq B$
- $P_{bc} \rightsquigarrow B ? P_{bc}$
- $P_{cd} \rightsquigarrow B \leq P_{cd}$



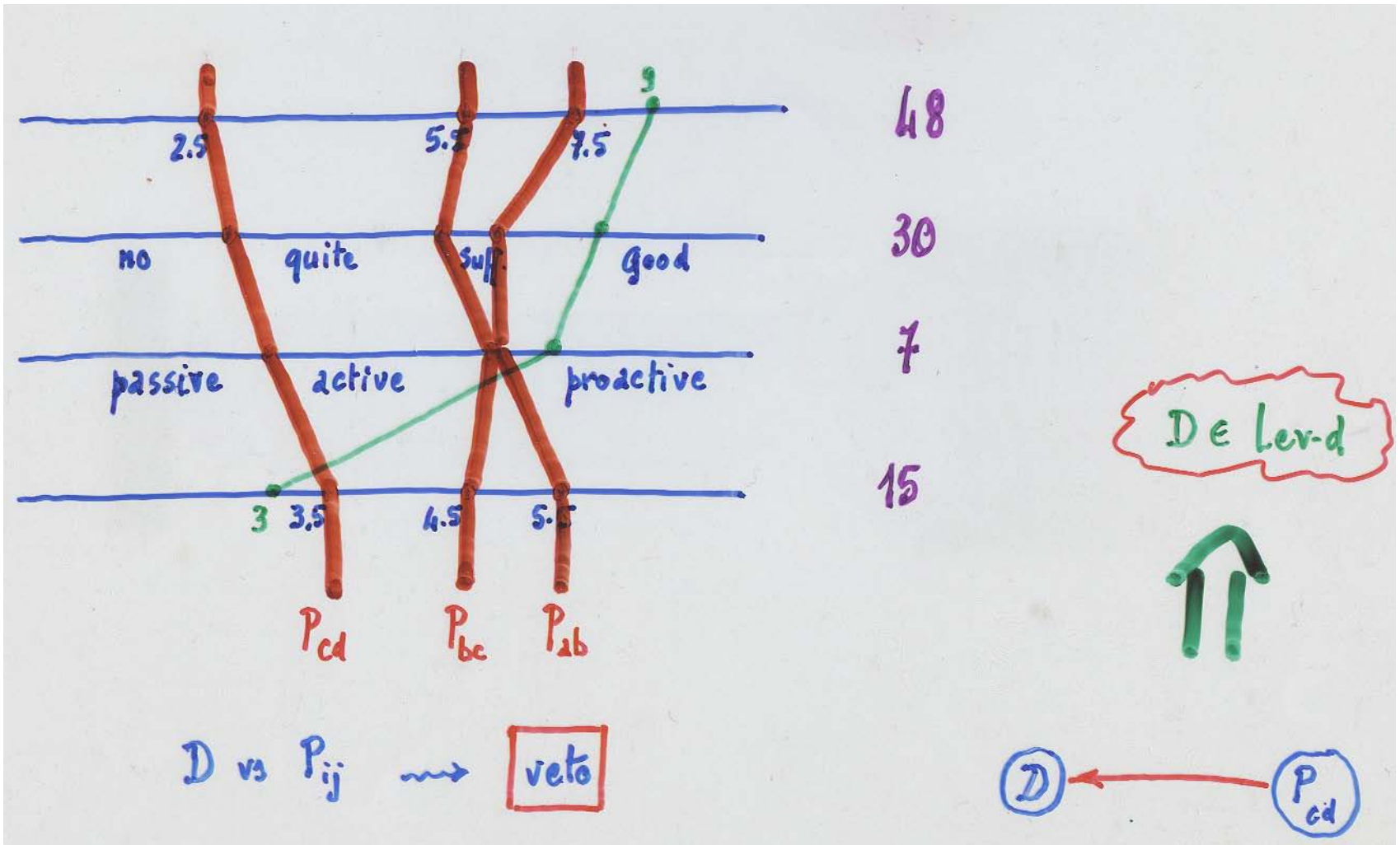
Student C





C vs $P_{ab} \rightsquigarrow P_{ab} S C$
 C vs $P_{bc} \rightsquigarrow$ it is $\begin{cases} C S P_{bc} \\ P_{bc} S C \end{cases}$



Student D



0. Data → the categories-levels (4) and the objects (40)
 - Choice of indicators-criteria (4)
 - Choice of weights (48, 30, 7, 15)
 - Definition of profiles (profiles = levels -1)
 - Definition of relations between K and P_{ij} (threshold, veto, ...)
 - Performances of students (A, B, C, D, ...)
 - Comparison between K and P_{ij}  rating of K

- The French school → Electre methods
- The main idea → outranking → reasons  pro (strong)
con (weak)
- Concordance, discordance, veto
(high) (low) (no)
- Four cases: $A \rightarrow B$, $A \leftarrow B$, $A \rightleftarrows B$, $A ? B$
- The importance of incomparability
- Threshold (α , β , veto) and sensitivity → what happens if α decreases?
- Subjectivity (where?)

Winning coalition (more...)

- Three parties →

Left	48%
Center	3%
Right	49%

- Threshold = $50\% + \varepsilon$
- What is the power of each party?

- Coalitions:

L	C	R	%	
0	0	0	0	
0	0	1	49	
0	1	0	3	
0	1	1	52 *	
1	0	0	48	
1	0	1	97 *	
1	1	0	51 *	
1	1	1	100	

* = minimal coalition

$p_L = p_C = p_R = \frac{1}{3}$

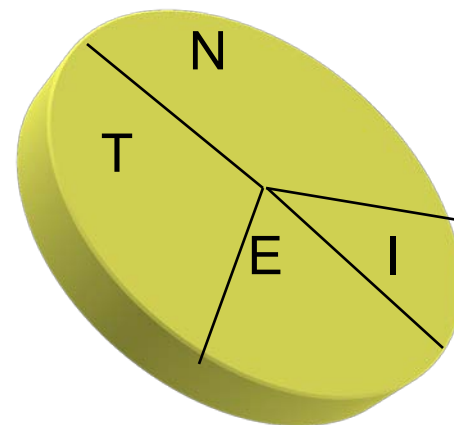
⇓

the same power



The groups:

- North (N) 39%
- Irish (I) 10%
- Editors (E) 21%
- Telefonica (T) 30%

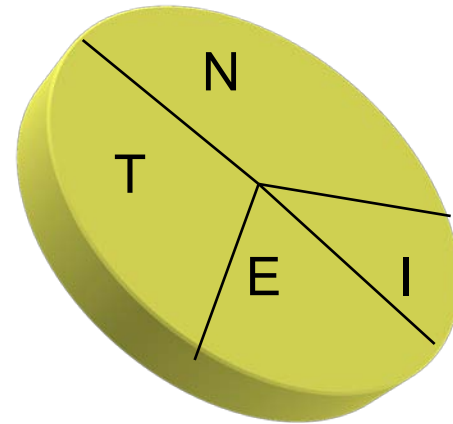


Coalitions...



Telecom

... of who with who ?



- **Winning coalition (WC)**: a coalition between some DM that permits the governance (that means the coalition overcomes the fixed threshold, usually $50\% + \epsilon$)
- **Critical WC (CWC)**: a WC in which the defection of some DM – but only some – doesn't permit the governance
- **Swing vote (SW)**: in a CWC a SW is a vote that, if modified, determine the failure of the coalition (that means the impossibility of governance).

See also:

Shapley index (1953)

Banzhaf index (1965)

Holler index (public goods, 1982)

N → 39%, I → 10%, E → 21%, T → 30%

Threshold = 50% + ε

N	I	E	T	%	Coa liz.			N	I	E	T	%	Coa liz.
<u>39</u>	<u>10</u>	<u>21</u>	<u>30</u>					<u>39</u>	<u>10</u>	<u>21</u>	<u>30</u>		
0	0	0	0	0	--			1	0	0	0	39	--
0	0	0	1	30	--			1	0	0	1	69	CVC
0	0	1	0	21	--			1	0	1	0	60	CVC
0	0	1	1	51	CVC			1	0	1	1	90	(CV)
0	1	0	0	10	--			1	1	0	0	49	--
0	1	0	1	40	--			1	1	0	1	79	CVC
0	1	1	0	31	--			1	1	1	0	70	CVC
0	1	1	1	61	CVC			1	1	1	1	100	(CV)

N → ...,

I → ...,

E → ...,

T → ...

Seminar of M. Henig

Decision Making Processes: Goals and Paradigms

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Exercise: How are they related to **DESIGN** processes?

