

Bi-oriented Graphs and Four Valued Logics

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Abstract

In this paper we show the relations between 4-valued logics (and more precisely of the DDT logic) and the use of bi-oriented graphs. Further on we focus on the use of bi-oriented graphs for non conventional preference modelling. More specifically we show how bi-oriented graphs can be used in order to represent extended preference structures of the type definable using the DDT logic (which has been created with the purpose of modelling hesitation in preference statements). We then study how transitive closure can be extended within such extended preference structures.

Keywords: DDT logic, bi-oriented graphs, signed graphs, transitive closure

1. Introduction

The DDT logic [9], [10], [23], [28], is a first order four valued logic (a logic accepting 4 values i.e. true, false, true and false, neither true nor false, epistemic states), an extension of the Belnap's logic [3], [4], including a weak negation. In this logic, negation does not coincide with complementation and the reasons for which an expression can be regarded as true are not complementary to the reasons for which it can be regarded as false. Therefore it could be seen as a logic about uncertainty and hesitation. The principal idea introduced by Belnap was to define a logic where the truth values are partially ordered on a bi-lattice.

Bi-oriented graphs were introduced by Tutte [29]. A bi-oriented graph is a graph, where each edge is regarded as a set of two half-edges, each half-edge of the graph being equipped with a sign + or -. This concept was already studied for a long time in the theory of homology and algebraic topology [15]. In the 50s the combinatorial aspects of bi-oriented graphs were studied by Harary [13] who defined in 1953 the notion of signed graph (see also [30]).

Graphs have been extensively used (among others) as a language for preference modelling (see [21]), including some cases of non conventional preference structures, using valued graphs (see [14]; for a general survey see [19]). On the other hand the DDT logic has been explicitly conceived as a language for preference modelling as well, a language aiming at capturing hesitation and qualitative uncertainty (see [25]) when decision makers express their preferences. However, until today there has been no graph representation of the preference structures allowed by the DDT language. It

turned out natural to consider bi-oriented graphs as an appropriate mean to fill this gap. Our paper aims at presenting exactly how these two languages match and how this can improve our preference modelling and decision support capability.

The paper is organised as follows. In Section 2 we briefly introduce four valued logics as well as the specific language DDT which is a first order language of this type. We also show how this language is used for preference modelling purposes (actually it has been developed for this reason). In Section 3 we introduce the principal elements of bi-oriented graphs. In Section 4 we show the existing connections between DDT logic and bi-oriented graphs by just extending the latter to directed graphes. We show, however, that the definition of transitive closure, as conceived in the classical theory of signed graphs is not appropriate for preference modelling purposes. In Section 5 we introduce some new types of transitive closure which form a complete set of rational transitivity. In Section 6 we generalise the notion of transitive closure and show where and how this applies. In Section 7 we introduce the concept of flow upon directed bi-oriented graphs and the consequences it may introduce. In Section 8 we show the application of the previously introduced concepts to the transitive closure of a whole graph. We conclude with some remarks and further research directions.

2. The DDT logic (four-valued logic)

2.1. Generalities

Belnap's original proposition ([3]) aimed at capturing situations where hesitation in establishing the truth of a sentence can be associated either to ignorance (poor information) or to contradiction (excess of information). In order to distinguish these two types of uncertainty, he suggested the use of four values forming a bi-lattice. The DDT logic ([23]) is a four-valued first order language extending Belnap's logic in two ways:

- introducing a weak negation which allows to establish a Boolean algebra (an idea inspired to the work of Dubarle; see [11]);
- introducing first order semantics, thus allowing to work with variables.

The language is based on a net distinction between the "negation" (which represents the part of the universe of discourse verifying a negated predicate and the "complement" (which represents the part of the universe which does not verify a predicate) since the two concepts do not necessarily coincide. The four values t (true), f (false), u (unknown) and k (contradiction), capture four epistemic states derived from the presence of information supporting or not a certain sentence. If α is a sentence then:

- α is true (t): there is evidence supporting α and there is no evidence against it;
- α is false (f): there is no evidence supporting α and there is evidence against it;
- α is unknown (u): there is neither evidence supporting α nor against it;
- α is contradictory (k): there is both evidence supporting α and against it.

The differences between the strong negation (\neg), the complement (\sim) and the weak negation (\approx) are presented in Table 1. The reader will note the Boolean algebra properties this structure allows. It is easy to check that $\sim \alpha \equiv \neg \approx \neg \approx \alpha$. Binary connectives are established using the usual Boolean algebra principle (conjunction being the *glb* and disjunction being the *lub* on the bi-lattice of the truth values; for details see [23]).

α	$\neg\alpha$	$\sim\alpha$	$\approx\alpha$	$\sim\sim\alpha$	$\neg\approx\alpha$	$\neg\sim\approx\alpha$	$\neg\sim\alpha$
t	f	f	k	u	k	u	t
k	k	u	t	f	f	t	u
u	u	k	f	t	t	f	k
f	t	t	u	k	u	k	f

Table 1: The truth tables of \sim , \approx and \neg and their combinations

We give now the definition of some strong monadic operators enabling to obtain "non contradictory" (only true or false) statements for a sentence α .

Definition 2.1.

$\mathbf{T}\alpha \equiv \alpha \wedge \sim \neg\alpha$: α is true

$\mathbf{K}\alpha \equiv \approx\alpha \wedge \approx \neg\alpha$: α is contradictory

$\mathbf{U}\alpha \equiv \neg\approx\alpha \wedge \neg\approx \neg\alpha$: α is unknown

$\mathbf{F}\alpha \equiv \neg\alpha \wedge \sim\alpha$: α is false

$\Delta\alpha \equiv \mathbf{T}\alpha \vee \mathbf{K}\alpha$: there is presence of truth in claiming α

$\Delta\neg\alpha \equiv \mathbf{F}\alpha \vee \mathbf{K}\alpha$: there is presence of truth in claiming $\neg\alpha$

$\neg\Delta\alpha \equiv \mathbf{F}\alpha \vee \mathbf{U}\alpha$: there is no presence of truth in claiming α

$\neg\Delta\neg\alpha \equiv \mathbf{T}\alpha \vee \mathbf{U}\alpha$: there is no presence of truth in claiming $\neg\alpha$

Obviously we get:

$\mathbf{T}\alpha \equiv \Delta\alpha \wedge \neg\Delta\neg\alpha$

$\mathbf{F}\alpha \equiv \neg\Delta\alpha \wedge \Delta\neg\alpha$

$\mathbf{U}\alpha \equiv \neg\Delta\alpha \wedge \neg\Delta\neg\alpha$

$\mathbf{K}\alpha \equiv \Delta\alpha \wedge \Delta\neg\alpha$

2.2. DDT and preference modelling

As already mentioned, the DDT logic has been conceived as a language aiming at capturing hesitation when preference statements need to be considered in decision making settings (see [1], [8], [12], [17], [20], [24], [25], [26], [27]).

The basic idea is simple. Consider the typical binary relation used in preference modelling: $S(x, y)$, to be read as " x is at least as good as y ". Given a set A (on which S applies), we can define a universe of discourse $A \times A$ for the predicate S . If now we allow the interpretations of S in $A \times A$ to be four valued, instead of binary valued as in conventional preference modelling, we obtain a more rich preference modelling language where:

- hesitation about a preference statement can be explicitly considered (for instance $\Delta S(x, y)$ will stand for "there is presence of truth in claiming that x is at least as good as y " or that there are sufficient positive reasons to claim it, see [25]);
- it is possible to construct richer preference structures beyond the well known $\langle P, I, J \rangle$ (preference, indifference, incomprability) ones, allowing for explicit preference relations about conflicting preferences, ignorance about preference etc. (see [26]);

- it is possible to give new and/or more elegant proofs for representation theorems allowing for numerical representations for interval preference structures (see [24], [27]);
- it is possible to conceive new procedures aiming at exploiting such rich preference structures in order to produce a recommendation (see [12], [17], [20]).

With respect to this framework, the reader will note that many of the representation theorems as well as many of the decision support procedures explicitly need to consider extended notions of transitivity, either well known ones such as “semi-transitivity” and “Ferrers” ([19]), or new ones ([16], [18], [24]). Under such a perspective a problem still open in the relevant literature concerns the extension and/or generalisation of the concept of transitive closure, a key issue in many decision support procedures. However, this calls for extending graph theory in order to be able to take into account the new preference structures the DDT logic allows. For this purpose it turned out natural to consider bi-oriented graphs as an appropriate extension of graph theory functional to the DDT language.

3. Bi-oriented graphs

Bi-oriented graphs were introduced by Tutte [29]. Then they were studied by several researchers ([7], [15], [31]), interested in the study of the flows in the bi-oriented graphs, but the notations and the results used in this paper are those used by Bessouf (see [5] and [6]), in which we studied the notion of paths, connectivity and transitive closure.

Consider an undirected graph $G = (V, E)$ (V being the set of vertices and E being the set of edges). The set of the half-edges of G is a set $\Phi(G)$ defined as follows:

$$\Phi(G) = \{(e, x) \in E \times V / e \text{ is incident to } x\}$$

Thus, each edge e between any two nodes x and y is represented by its two half-edges (e, x) and (e, y) .

Definition 3.1. [5], [6]. A bi-orientation of G is a signature of its half-edges

$$\tau : \Phi(G) \mapsto \{-1, +1\}$$

It is agreed that $\tau(e, x) = 0$ if (e, x) is not a half-edge of G , (which makes possible to extend τ with any $(E \times V)$, which we will do henceforth).

A bi-oriented graph is a graph provided with a bi-orientation τ , denoted as $G_\tau = (V, E; \tau)$ or simply (if there is no ambiguity) by $G_\tau = (V, E)$.

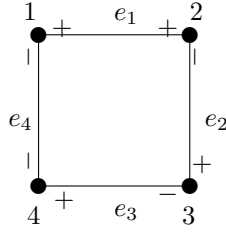
The **Four** possible bi-orientations of an edge $\{x, y\}$ of G_τ are (we replace +1 with + and -1 with -) shown in Figure 1:



Figure 1: Possible bi-orientations

The bi-oriented edges $x \bullet^+ \text{---} \bullet^+ y$ and $x \bullet^- \text{---} \bullet^- y$ are called *opposed to each other*. Same applies for $x \bullet^+ \text{---} \bullet^- y$ and $x \bullet^- \text{---} \bullet^+ y$

Example. Let $G_\tau = (V, E)$ be a bi-oriented graph as follows $V = \{1, 2, 3, 4\}$, $E = \{e_1, e_2, e_3, e_4\}$ and $\tau(e_1, 1) = \tau(e_1, 2) = +1$, $\tau(e_2, 2) = -\tau(e_2, 3) = -1$, $\tau(e_3, 4) = -\tau(e_3, 3) = +1$, $\tau(e_4, 1) = \tau(e_4, 4) = -1$



Definition 3.2. Let $G_\tau = (V, E)$ be a bi-oriented graph.

- **Reorienting** an edge, means replacing its bi-orientation by its opposite.

the reorientation of $x \bullet^- \text{---} \bullet^+ y$ is $x \bullet^+ \text{---} \bullet^- y$
the reorientation of $x \bullet^+ \text{---} \bullet^- y$ is $x \bullet^- \text{---} \bullet^+ y$
the reorientation of $x \bullet^- \text{---} \bullet^- y$ is $x \bullet^+ \text{---} \bullet^+ y$
the reorientation of $x \bullet^+ \text{---} \bullet^+ y$ is $x \bullet^- \text{---} \bullet^- y$

- **Rotation** of an edge, means to permute the two signatures of its half-edges.

the rotation of $x \bullet^- \text{---} \bullet^+ y$ is $x \bullet^+ \text{---} \bullet^- y$
the rotation of $x \bullet^+ \text{---} \bullet^- y$ is $x \bullet^- \text{---} \bullet^+ y$
the rotation of $x \bullet^- \text{---} \bullet^- y$ is $x \bullet^+ \text{---} \bullet^+ y$
the rotation of $x \bullet^+ \text{---} \bullet^+ y$ is $x \bullet^- \text{---} \bullet^- y$

• **Left $\{x, y\}$ -switching** (resp. **Right $\{x, y\}$ -switching**) an edge $\{x, y\}$, means replacing the signature of its left -close to x - (resp. right -close to y) half edge by its opposite.

the left $\{x, y\}$ -switching of $x \bullet^- \text{---} \bullet^+ y$ is $x \bullet^+ \text{---} \bullet^+ y$
the left $\{x, y\}$ -switching of $x \bullet^+ \text{---} \bullet^- y$ is $x \bullet^- \text{---} \bullet^- y$
the left $\{x, y\}$ -switching of $x \bullet^- \text{---} \bullet^- y$ is $x \bullet^+ \text{---} \bullet^- y$

the left $\{x, y\}$ -switching of $x \bullet \overset{+}{\text{---}} \overset{+}{\text{---}} \bullet y$ is $x \bullet \overset{-}{\text{---}} \overset{+}{\text{---}} \bullet y$

the right $\{x, y\}$ -switching of $x \bullet \overset{-}{\text{---}} \overset{+}{\text{---}} \bullet y$ is $x \bullet \overset{-}{\text{---}} \overset{-}{\text{---}} \bullet y$
the right $\{x, y\}$ -switching of $x \bullet \overset{+}{\text{---}} \overset{-}{\text{---}} \bullet y$ is $x \bullet \overset{+}{\text{---}} \overset{+}{\text{---}} \bullet y$
the right $\{x, y\}$ -switching of $x \bullet \overset{-}{\text{---}} \overset{-}{\text{---}} \bullet y$ is $x \bullet \overset{-}{\text{---}} \overset{+}{\text{---}} \bullet y$
the right $\{x, y\}$ -switching of $x \bullet \overset{+}{\text{---}} \overset{+}{\text{---}} \bullet y$ is $x \bullet \overset{+}{\text{---}} \overset{-}{\text{---}} \bullet y$

If τ is a bi-orientation of G_τ , and if F is a part of E then, applying any of the above operations to F means that we replace the bi-orientation of each edge of F by the result of these, the bi-orientations of the other edges remaining unchanged.

Example. The bi-oriented graph G_{τ_F} is obtained from G_τ by reorientation of F . See Figure 2.

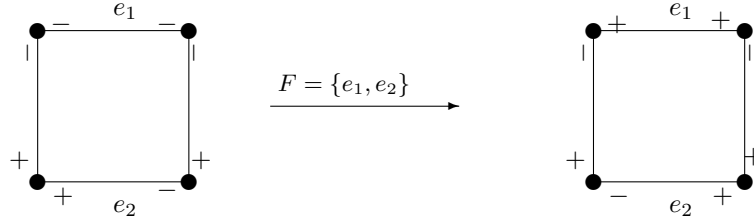


Figure 2: Reorientation of F

Definition 3.3. [5], [6]. Let $G_\tau = (V, E)$ be a bi-oriented graph and W (resp. \overline{W}) be a function defined on V (resp. E) as follows:

$$W : V \rightarrow \mathbb{Z} \text{ such that: } W(x) = \sum_{e \in E} \tau(e, x)$$

$$\overline{W} : E \rightarrow \{-2, 0, 2\} \text{ such that: } \overline{W}(e) = \sum_{x \in V} \tau(e, x)$$

An edge e of E is called a positive (resp. negative) edge, if $\overline{W}(e) = 0$ (resp. $\overline{W}(e) = \pm 2$).

G_τ is called all positive (resp. all negative) bi-oriented graph, if $\forall e \in G_\tau : \overline{W}(e) = 0$ (resp. $\overline{W}(e) = \pm 2$).

An elementary cycle C in G_τ is called a negative cycle, if the number of its edges such that $\overline{W}(e) = \pm 2$ is odd.

Definition 3.4. [5],[6]. Let $G_\tau = (V, E)$ be a bi-oriented graph, and let P be a (not necessarily elementary) chain connecting x and y in G_τ :

$$P : x e_1 x_1 \dots e_i x_{i+1} e_{i+1} \dots x_{k-1} e_k y.$$

(x, x_1, \dots, y are vertices of G_τ and e_1, e_2, \dots, e_k are edges of G_τ).

Assume that: $\tau(e_1, x) = \alpha$ and $\tau(e_k, y) = \beta$ such that $\alpha, \beta \in \{-1, +1\}$, and for every vertex $x_i \in V(P)$ we put, $W_P(x_i) = \tau(e_i, x_i) + \tau(e_{i+1}, x_i)$ hence we denote:

$$P_{(\alpha,\beta)}(x,y) : x^\alpha e_1 x_1 \dots e_i x_{i+1} e_{i+1} \dots x_{k-1} e_k y^\beta.$$

$P_{(\alpha,\beta)}(x,y)$ is called a **b-path** from x^α to y^β if:

- (i) $k \geq 1$.
- (ii) $\tau(e_1, x) = \alpha$, and $\tau(e_k, y) = \beta$ ($\alpha = \beta$ being possible).
- (iii) $W_P(x_i) = 0, \forall i = 1 \dots k-1$ and $k > 1$.
- (iv) $P_{(\alpha,\beta)}(x,y)$ is minimal for the property (i)-(iii).

Examples of b-paths can be seen in Figure 3.

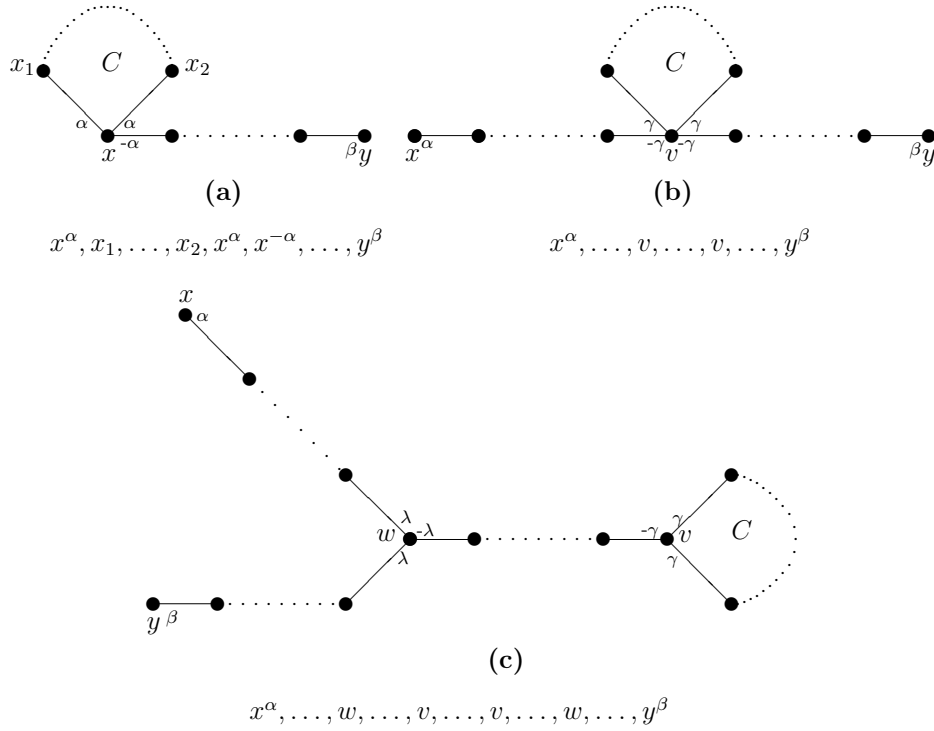


Figure 3: The three types of b-paths on bi-oriented graphs where C is a negative elementary cycle

Remark: If $P_{(\alpha,\beta)}(x,y)$ is a b-path from x^α to y^β , then $P_{(\beta,\alpha)}(y,x)$ is also a b-path from y^β to x^α , such that: $P_{(\beta,\alpha)}(y,x) : y^\beta e_k x_{k-1} \dots e_{i+1} x_{i+1} e_i \dots x_1 e_1 x^\alpha$.

Definition 3.5. (Transitive Closure in Bi-oriented Graphs)

Let $G_\tau = (V, E)$ be a bi-oriented graph with $|V| \geq 3$. The transitive closure of G_τ is the graph denoted $Ft(G_\tau) = (V, Ft(E), \tau)$ such that $\{x^\alpha, y^\beta\} \in Ft(E)$ if there is a b-path $P_{(\alpha,\beta)}(x,y)$ from x^α to y^β in G_τ (x and y are not necessarily distinct; see example in figure 4).

Corollary 3.6. G_τ is a partial graph of $Ft(G_\tau)$.

Proof. It is obvious from the definition. ■

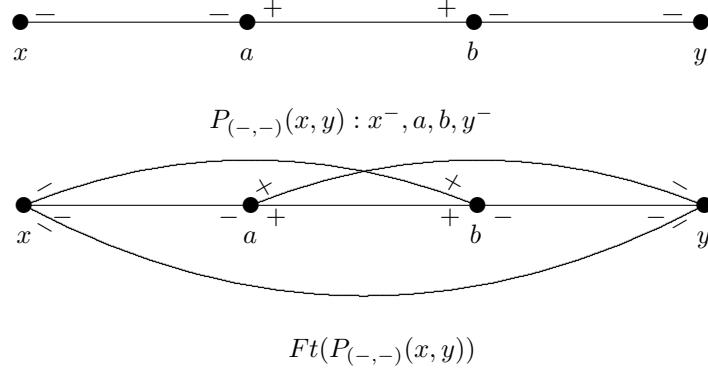


Figure 4: An example of transitive closure of bi-oriented graph

4. The relations between bi-oriented graphs and the four valued logic (DDT logic)

Let A be a discrete countable set and let S be the binary relation “at least as good as” applied upon A . The four strong monadic operators on S , $TS(x, y)$, $FS(x, y)$, $US(x, y)$ and $KS(x, y)$, are defined as follows:

TS(x,y): there exist sufficient positive reasons to establish $S(x, y)$ and there are not enough negative reasons to establish $\neg S(x, y)$; $S(x, y)$ is true

FS(x,y) : there do not exist sufficient positive reasons to establish $S(x, y)$ and there exist enough negative reasons to establish $\neg S(x, y)$; $S(x, y)$ is false.

US(x,y): there do not exist sufficient positive reasons to establish $S(x, y)$ and there are not enough negative reasons to establish $\neg S(x, y)$; $S(x, y)$ is unknown.

KS(x,y): there exist sufficient positive reasons to establish $S(x, y)$ and sufficient negative reasons to establish $\neg S(x, y)$; $S(x, y)$ is contradictory.

More formally, we accept that S and $\neg S$ are not complementary and they do not cover the whole set of possible situations. We can express this idea by introducing the sentence $\Delta S(x, y)$:

- $\Delta S(x, y)$: there is presence of truth in claiming that x is at least as good as y (presence of positive reason)
- $\Delta \neg S(x, y)$: there is presence of truth in claiming that x is not at least as good as y (presence of negative reason)
- $\neg \Delta S(x, y)$: there is no presence of truth in claiming that x is at least as good as y (absence of positive reason)
- $\neg \Delta \neg S(x, y)$: there is no presence of truth in claiming that x is not at least as good as y (absence of negative reason)

Consequently we have:

$$TS(x, y) \Leftrightarrow \Delta S(x, y) \wedge \neg \Delta \neg S(x, y)$$

$$FS(x, y) \Leftrightarrow \neg \Delta S(x, y) \wedge \Delta \neg S(x, y)$$

$$US(x, y) \Leftrightarrow \neg \Delta S(x, y) \wedge \neg \Delta \neg S(x, y)$$

$$KS(x, y) \Leftrightarrow \Delta S(x, y) \wedge \Delta \neg S(x, y)$$

We are ready now to show a first intuitive application of bi-oriented graphs to this type of preference structures. We first need to extend the theory of bi-oriented graphs to directed graphs (which is straightforward). We then add to Tsoukias and Vincke 's [18] results a graphic representation given as follows: Let $G_\tau = (V, E)$ be a directed bi-oriented graph and S be the binary relation given above defined on V . $\forall x, y \in V$ we put:

- $\Delta S(x, y) \rightarrow \tau(e, x) = +1$
- $\neg \Delta S(x, y) \rightarrow \tau(e, x) = -1$
- $\Delta \neg S(x, y) \rightarrow \tau(e, y) = +1$
- $\neg \Delta \neg S(x, y) \rightarrow \tau(e, y) = -1$

Henceforth, a directed bi-oriented graph provided with the relation S is noted (G_τ, S) . The graphic representation of the 4 operators $TS(x, y)$, $FS(x, y)$, $KS(x, y)$

and $US(x, y)$ is given in figure 5, the orientation of the edges means that the relation $S(x, y)$ is from x to y :



Figure 5: Signed graphs of TS , FS , KS , US from x to y

The reader should note that the graph, being directed, could also admit the inverse edges (from y to x ; figure 6)



Figure 6: bi-oriented graphs of TS , FS , KS , US from y to x

The result is that in order to represent the strict preference relation (according to [25]) we need a graph such as in figure 7. Equally, for the weak preference relation we need a graph such as in figure 8.

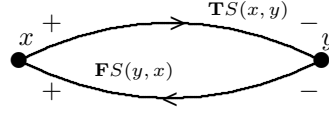


Figure 7: x is strictly preferred to y

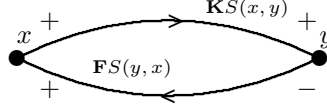


Figure 8: x is weakly preferred to y

Let ϕ be a predicate admitting a representation through a bi-oriented graph. We indicate by:

- $H_r(\phi)$ (resp. $H_l(\phi)$) the right switching (resp. left switching) of ϕ .
- $R(\phi)$ the reorientation of ϕ .
- $T(\phi)$ the rotation of ϕ .

Proposition 4.1. *Let $\psi \in \{\mathbf{T}, \mathbf{F}, \mathbf{K}, \mathbf{U}\}$ and S be the usual relation, then :*

1. $H_r(\psi(S)) = \psi(\sim S)$
2. $H_l(\psi(S)) = \psi(\sim \sim S)$
3. $R(\psi(S)) = \psi(\sim S)$
4. $T(\psi(S)) = \psi(\neg S)$

Proof. From Definition 2.1 it is easy to show that for any given formula ϕ , $\mathbf{T} \sim \phi \equiv \mathbf{K}\phi$, $\mathbf{T} \sim \phi \equiv \mathbf{F}\phi$, $\mathbf{K} \sim \phi \equiv \mathbf{U}\phi$, $\mathbf{K} \sim \phi \equiv \mathbf{T}\phi$ and so on (we omit the whole set of equivalences for space reasons).

Applying the definition of H_r , H_l , R and T of α from Definition 3.2 and from figure 5 we complete the proof. ■

The result can be shown in the following table.

ψ	$H_r(\alpha)$	$H_l(\alpha)$	$R(\alpha)$	$T(\alpha)$
TS	KS	US	FS	FS
FS	US	KS	TS	TS
KS	TS	FS	US	KS
US	FS	TS	KS	US

Proposition 4.2. *The conventional transitive closure of bi-oriented graphs is associative.*

Proof. Let $e_1 = \{x^\alpha, y^\beta\}$, $e_2 = \{y^{-\beta}, z^\gamma\}$ and $e_3 = \{z^{-\gamma}, w^\lambda\}$, be three edges of G_τ . The transitive closure of the b-path $P_{(\alpha, \beta)}(x, z) : x^\alpha e_1 e_2 z^\gamma$; is the edge $e_4 = \{x^\alpha, z^\gamma\}$. The transitive closure of the b-path $P_{(\alpha, \lambda)}(x, w) : x^\alpha e_4 e_3 w^\lambda$; is the edge $e_5 = \{x^\alpha, w^\lambda\}$. The transitive closure of the b-path $P_{(-\beta, \lambda)}(y, w) : y^{-\beta} e_2 e_3 w^\lambda$; is the edge $e_4 = \{y^{-\beta}, w^\lambda\}$. And the transitive closure of the b-path $P_{(\alpha, \lambda)}(x, w) : x^\alpha e_1 e_4 w^\lambda$; is the edge $e_5 = \{x^\alpha, w^\lambda\}$.

It is the same if a b-path does not exist between at least two vertices, because in this case the transitive closure does not exist. Hence, the transitive closure is associative. ■

Proposition 4.3. Let (G_τ, S) be a bi-oriented graph provided with the binary relation S , such that $G_\tau = (A, E)$. Adopting the operation of transitive closure in $Ft(G_\tau)$ as introduced in Definition 3.5 $\forall x, y$ and $z \in A$ we get the following:

$$\begin{aligned}
& \mathbf{TS}(x, y) \wedge \mathbf{TS}(y, z) \rightarrow \mathbf{TS}(x, z) \\
& \mathbf{TS}(x, y) \wedge \mathbf{KS}(y, z) \rightarrow \mathbf{KS}(x, z) \\
& \mathbf{FS}(x, y) \wedge \mathbf{FS}(y, z) \rightarrow \mathbf{FS}(x, z) \\
& \mathbf{FS}(x, y) \wedge \mathbf{US}(y, z) \rightarrow \mathbf{US}(x, z) \\
& \mathbf{US}(x, y) \wedge \mathbf{TS}(y, z) \rightarrow \mathbf{US}(x, z) \\
& \mathbf{US}(x, y) \wedge \mathbf{KS}(y, z) \rightarrow \mathbf{FS}(x, z) \\
& \mathbf{KS}(x, y) \wedge \mathbf{FS}(y, z) \rightarrow \mathbf{KS}(x, z) \\
& \mathbf{KS}(x, y) \wedge \mathbf{US}(y, z) \rightarrow \mathbf{TS}(x, z)
\end{aligned}$$

Proof. According to the definition of the transitive closure of the bi-oriented graphs, and the graphic interpretation of the 4 operators $\{\mathbf{TS}, \mathbf{FS}, \mathbf{KS}, \mathbf{US}\}$, the proof is given by the graphs in figure 9. ■

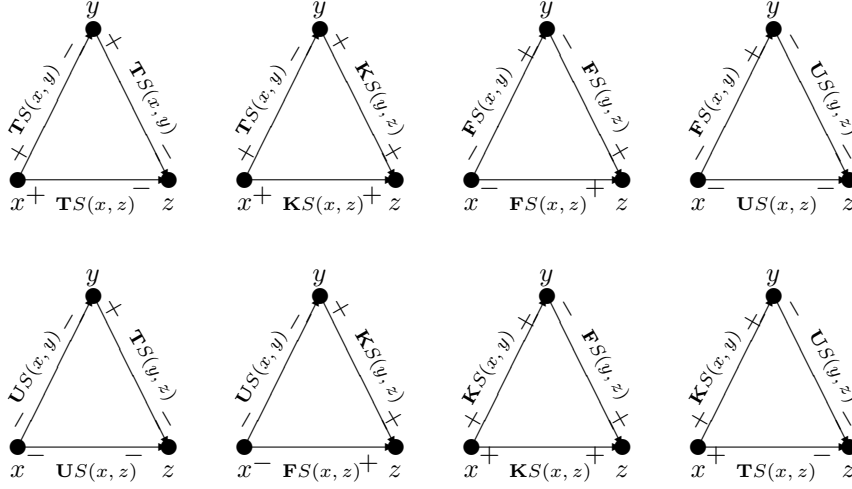


Figure 9: Graphical demonstration of Proposition 4.3

Remark: Note that the implications shown in Proposition 4.3 are not symmetric with respect to the left-hand components of the implication. For instance, $\mathbf{TS}(x, y) \wedge \mathbf{KS}(y, z) \rightarrow \mathbf{KS}(x, z)$ holds, however the transitive closure does not imply $\mathbf{KS}(x, y) \wedge \mathbf{TS}(y, z) \rightarrow \mathbf{KS}(x, z)$. We think that such a symmetry of the left-hand components is necessary and coherent with the semantic of the preference modelling. For this reason, in the next section, we will propose some new interpretations and definitions of transitivities of positive and negative reasons. Our propositions will be motivated from preference modelling point of view and will use directed bi-oriented graph modelling.

5. New Transitive Closures

As it is commented at the end of the last section, we will introduce a number of transitivity operations based on the DDT language, satisfying all of them the symmetry on the left-hand components of the transitivity.

1. $\Delta S(x, y) \wedge \Delta S(y, z) \rightarrow \Delta S(x, z)$ (transitivity of positive reasons: ΔS)
2. $\neg \Delta S(x, y) \wedge \neg \Delta S(y, z) \rightarrow \neg \Delta S(x, z)$ (negative transitivity of positive reasons: ΔS)

The above transitive closures represent how transitivity applies to the positive part of the preference modelling reasoning. Substituting to ΔS , $\Delta \neg S$ we get the equivalent transitive closure for the negative part of the preference modelling reasoning.

3. $\Delta \neg S(x, y) \wedge \Delta \neg S(y, z) \rightarrow \Delta \neg S(x, z)$ (transitivity of negative reasons: $\Delta \neg S$)
4. $\neg \Delta \neg S(x, y) \wedge \neg \Delta \neg S(y, z) \rightarrow \neg \Delta \neg S(x, z)$ (negative transitivity of negative reasons: $\Delta \neg S$)

These four transitive closures combine the same type of information (presence or absence of positive or negative reasons), hence they respect the symmetry condition that we are looking for. We further introduce four transitive closures, here by shown as cases 5, 6, 7 and 8 where we combine positive and negative reasons aiming at creating (positive or negative) reasons. Such a definition being not symmetric on the left-hand components of the transitivity, we will impose the symmetry :

5. $\Delta S(x, y) \wedge \neg \Delta \neg S(y, z) \rightarrow \Delta S(x, z)$ (creating positive reasons)
 $\neg \Delta \neg S(x, y) \wedge \Delta S(y, z) \rightarrow \Delta S(x, z)$
6. $\Delta \neg S(x, y) \wedge \neg \Delta S(y, z) \rightarrow \neg \Delta S(x, z)$ (eliminating positive reasons)
 $\neg \Delta S(x, y) \wedge \Delta \neg S(y, z) \rightarrow \neg \Delta S(x, z)$
7. $\Delta \neg S(x, y) \wedge \neg \Delta S(y, z) \rightarrow \Delta \neg S(x, z)$ (creating negative reasons)
 $\neg \Delta S(x, y) \wedge \Delta \neg S(y, z) \rightarrow \Delta \neg S(x, z)$
8. $\Delta S(x, y) \wedge \neg \Delta \neg S(y, z) \rightarrow \neg \Delta \neg S(x, z)$ (eliminating negative reasons)
 $\neg \Delta \neg S(x, y) \wedge \Delta S(y, z) \rightarrow \neg \Delta \neg S(x, z)$

The eight transitive closures are represented graphically in figure 10.

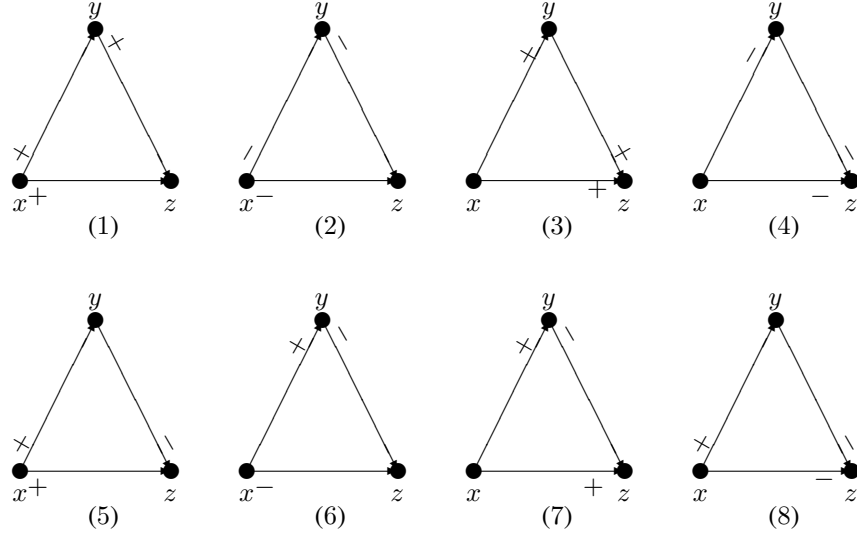


Figure 10: Graphical representation of the 8 transitive closures

Let's see now what happens if we assume any of the above 8 transitive closures holding. There are 255 of such combinations ($2^8 - 1$). The interested reader can check them in Annexe A (at the end of the paper). Table 2 presents the result when each of the above transitive closures holds alone. Each type of transitive closure is represented by its number; for instance the first table numbered 1 represents the transitivity of positive reasons. Table 3 presents the results when the four direct and the four undirect transitive closures hold simultaneously, while the case where all 8 closures hold simultaneously is presented in Table 4. The tables stand for sentences of the type $\alpha S(x, y) \wedge \alpha S(y, z)$ where $\alpha \in \{\mathbf{T}, \mathbf{K}, \mathbf{U}, \mathbf{F}\}$ (rows will stand for $S(x, y)$ and columns for $S(y, z)$).

1	TS	FS	KS	US
TS	ΔS	-	ΔS	-
FS	-	-	-	-
KS	ΔS	-	ΔS	-
US	-	-	-	-

2	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	-	-	-	-
US	-	$\neg\Delta S$	-	$\neg\Delta S$

3	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	-	$\Delta\neg S$	$\Delta\neg S$	-
US	-	-	-	-

4	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	-	-	-
KS	-	-	-	-
US	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$

5	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	-	-	-
KS	ΔS	-	-	ΔS
US	ΔS	-	ΔS	-

6	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	-	$\neg\Delta S$	-	$\neg\Delta S$
US	-	$\neg\Delta S$	$\neg\Delta S$	-

7	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	-	$\Delta\neg S$	-	$\Delta\neg S$
US	-	$\Delta\neg S$	$\Delta\neg S$	-

8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	-	-	-
KS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

Table 2: The eight basic transitive closures

1234	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	KS	-
US	$\neg\Delta\neg S$	$\neg\Delta S$	-	US

5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	-	-
US	TS	FS	-	-

Table 3: Applying transitive closures 1,2,3 and 4 or 5,6,7 and 8 simultaneously

12345678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	KS	-
US	TS	FS	-	US

Table 4: Combining all eight transitive closures

Discussion. First of all the reader should note that there are no other possible “rational” transitive closures we can define within this framework. The four direct transitive closures represent the natural extension of the notion of transitivity within our framework (symmetric combination of the presence or the absence of positive or negative reasons). The four undirect ones combine asymmetrically the presence (or absence) of positive (or negative reasons), but with a symmetric result (on the left-hand components). Analysing the different combinations of these transitive closures we can observe that:

- direct transitivity of **T**, **F**, **K**, **U** is obtainable only through the closures labelled 1,2,3 and 4;
- the closures labelled 5,6,7 and 8 only help in strengthening the conclusions of “hesitating combinations” such as $\mathbf{T}s(x, y) \wedge \mathbf{K}s(y, z)$; the result is always either **T** or **F**;
- all the tables are symmetric with respect to the diagonal.
- Transitive closures 5 and 6 have a special attitude when we have to combine unknown cases with contradictory ones. They provide contradictory conclusions. For instance, with (**K** and **U**), 5 implies ΔS while 6 implies $\neg \Delta S$. Hence, we conclude that if we impose 5 and 6 together, we will not have any conclusion for **K** and **U** (similarly for **U** and **K**). Because of a similar reasoning, there are no conclusion for (**K** and **U**) or (**U** and **K**) when 7 and 8 are imposed together.
- a more detailed analysis of the 255 combinations allows to reveal which are the minimal conditions in order to obtain a precise result (i.e. transitivity of ΔS or of $\mathbf{K}S$ etc.).

6. Generalisation of Transitive Closure

Consider a directed bi-oriented graph $G_t(V, E)$ and let us denote by x_i any element of V . The usual definitions of the signatures of bi-oriented graphs hold.

Definition 6.1. *we define as t -path any sequence of directed bi-oriented edges which does not contain a sequence x_{i-1}, x_i, x_{i+1} such that:*

- $\tau(x_{i-1}x_i, x_{i-1}) = 1$;
 - $\tau(x_{i-1}x_i, x_i) = -1$;
 - $\tau(x_ix_{i+1}, x_i) = -1$;
 - $\tau(x_ix_{i+1}, x_{i+1}) = 1$;
- or
- $\tau(x_{i-1}x_i, x_{i-1}) = -1$;
 - $\tau(x_{i-1}x_i, x_i) = -1$;
 - $\tau(x_ix_{i+1}, x_i) = +1$;
 - $\tau(x_ix_{i+1}, x_{i+1}) = +1$;

Remark: The edges of the sequence x_{i-1}, x_i, x_{i+1} in the Definition 6.1 are positive edges in the first case and are negative edges in the second case (see Definition 3.3). The reader will note that it does not make any sense to establish a transitive closure among edges which do not form a t -path (why to combine a true value with a false one?).

Definition 6.2. We define as transitive closure of any sequence of 2 edges $(x_{i-1}x_i, x_ix_{i+1})$, the sequence being a t -path within a directed bi-oriented graph, the establishment of an edge $x_{i-1}x_{i+1}$ such that any of the following conditions hold:

- $\tau(x_{i-1}x_i, x_{i-1}) = \tau(x_ix_{i+1}, x_i) = \tau(x_{i-1}x_{i+1}, x_{i-1})$
- $\tau(x_{i-1}x_i, x_i) = \tau(x_ix_{i+1}, x_{i+1}) = \tau(x_{i-1}x_{i+1}, x_{i+1})$
- $\tau(x_{i-1}x_{i+1}, x_{i-1}) = \max(\tau(x_{i-1}x_i, x_{i-1}), \tau(x_ix_{i+1}, x_{i+1}))$ or $\max(\tau(x_{i-1}x_i, x_i), \tau(x_ix_{i+1}, x_i))$
- $\tau(x_{i-1}x_{i+1}, x_{i+1}) = \min(\tau(x_{i-1}x_i, x_{i-1}), \tau(x_ix_{i+1}, x_{i+1}))$ or $\min(\tau(x_{i-1}x_i, x_i), \tau(x_ix_{i+1}, x_i))$

This definition generalises the eight transitive closures introduced in section 5 and described in figure 10.

Proposition 6.3. A transitive closure within a t -path does not always provide a t -path.

Proof. It is sufficient to consider a sequence of three edges from x to y to z to w such that $\tau(xy, x) = 1, \tau(xy, y) = -1, \tau(yz, y) = -1, \tau(yz, z) = -1, \tau(zw, z) = -1, \tau(zw, w) = 1$. The sequence $xyzw$ is a t -path, but establishing any transitive closure between x and z or between y and w (which are possible) will result in a sequence which is not a t -path. ■

Definition 6.4. We define as strong t -path (and we denote it a t_s -path) any t -path which remains such under any sequence of transitive closures.

Theorem 6.5. Let $G_t(V, E)$ be a directed all negative bi-oriented graph. Every t -path is a t_s -path.

Proof. According to the Definition 6.1, a b -path in all negative bi-oriented graph does not admit positive edges, and the definition of transitive closure described above of this t_s -path gives only one of the two edges of the above sequence. ■

Theorem 6.6. Let $G_t(V, E)$ be a directed all positive bi-oriented graph. Every t -path is a b -path, then it is a t_s -path.

Proof. According to the Definition 6.1, the edges of the t -path in all positive bi-oriented graph which is a b -path are of the same type. Then the edge of the transitive closure described above of this t_s -path is of the same type of this latter. ■

7. Preference flows in bi-oriented graphs

Consider a directed bi-oriented graph $G_t(V, E)$ and two vertices x and y in V . The graph being directed we need to consider explicitly the two different edges xy and yx . The result is that we need to consider four different signatures (the graph being bi-oriented): $\tau(xy, x), \tau(xy, y), \tau(yx, y), \tau(yx, x)$.

We denote as a flow between the vertices x and y the function $\varphi(xy) : V \times V \mapsto \{-4, -2, 0, 2, 4\}$ such that:

$\varphi(xy) = \tau(xy, x) - \tau(xy, y) - \tau(yx, y) + \tau(yx, x)$.
Clearly $\varphi(xy) = -\varphi(yx)$.

Given a pair of vertices x and y within a directed bi-oriented graph we can now distinguish three possible situations:

- strong asymmetry such that $\varphi(xy) = 4$ ($\varphi(yx) = -4$);
- weak asymmetry such that $\varphi(xy) = 2$ ($\varphi(yx) = -2$);
- symmetry such that $\varphi(xy) = 0$ ($\varphi(yx) = 0$);

This is coherent with the intuition ([26]) that preferences with no hesitation should “count more” with respect to preferences where the decision maker may have (for several different reasons) some hesitation. In case we consider a decision problem where a ranking is expected to be constructed out of a graph of preferences (which may include hesitation) this idea turns to become very useful since it allows to count differently strong asymmetric relations and weak asymmetric relations, allowing thus a more fine ranking of the set of alternatives (see also the method suggested in [12]).

8. Directed and undirected transitive Closures in the directed bi-oriented graphs

The transitive closure of bi-oriented graphs defined in Section 3, which is conditioned by the existence of a b-path is not symmetric with respect to left side of the implication. However, through the symmetric transitive closures defined for the DDT language in section 5, we can define transitive closures, which are called direct and undirect transitive closures for a directed bi-oriented graphs (G_τ, S) in which the notion of the b-path is not necessary. In the following (G_τ, S) is a directed bi-oriented graph provided with the relation S , $G_\tau = (V, E)$.

Proposition 8.1. *Let (G_τ, S) be a directed all positive bi-oriented graph (i.e, $\forall e \in E : \overline{W}(e) = 0$). The transitive closure of (G_τ, S) , is the graph denoted $(Ft(G_\tau), S) = (V, Ft(E))$ such that $e = \{x^\alpha, y^\beta\} \in Ft(E)$ if there exist a b-path $P_{(\alpha, \beta)}(x, y)$ in (G_τ, S) .*

Proof. The edges of $P_{(\alpha, \beta)}(x, y)$ are of the types **T** or **F**, and according to the directed transitive closures of **F** and **T** which are given in tables 3 and 4, we have: **T** \wedge **T** \rightarrow **T** and **F** \wedge **F** \rightarrow **F**, from where the result. ■

Corollary 8.2. (G_τ, S) is a partial graph of $(Ft(G_\tau), S)$.

Proof. It is obvious from the proposition. ■

Corollary 8.3. $(Ft(G_\tau), S)$ is a directed all positive bi-oriented graph, such that $\forall e \in Ft(E) : \overline{W}(e) = 0$.

Proof. The transitive closure of $P_{(\alpha, \beta)}(x, y)$ is the positive edge $e = \{x^\alpha, y^\beta\}$ which is of type **T** or **F**, and they are positive edges with $\overline{W}(\mathbf{F}) = \overline{W}(\mathbf{T}) = 0$. ■

Proposition 8.4. Let (G_τ, S) be a directed all negative bi-oriented graph such that $\forall e \in E : \overline{W}(e) = -2$. The transitive closure of (G_τ, S) , is the graph denoted $(Ft(G_\tau), S) = (V, Ft(E))$ such that $e = \{x^-, y^-\} \in Ft(E)$ if there exist a t_s -path, $Q(x^-, y^-) : x^- e_1 x_1 e_2 x_2 \dots x_k e_k y^-$ in (G_τ, S) .

Proof. According to the directed transitive closure of **U** given in tables 3 and 4 we have: $\mathbf{U} \wedge \mathbf{U} \rightarrow \mathbf{U}$, from where the result. ■

Corollary 8.5. (G_τ, S) is a partial graph of $(Ft(G_\tau), S)$.

Proof. It is obvious from the proposition. ■

Corollary 8.6. $(Ft(G_\tau), S)$ is a directed all negative bi-oriented graph, such that $\forall e \in Ft(E) : \overline{W}(e) = -2$.

Proof. The closure of $Q(x^-, y^-)$ is a negative edge $e = \{x^-, y^-\}$ which of type **U** and **U** is a negative edge with $\overline{W}(\mathbf{U}) = -2$. ■

Proposition 8.7. Let (G_τ, S) be a directed all negative bi-oriented graph such that $\forall e \in E : \overline{W}(e) = +2$. The transitive closure of (G_τ, S) , is the graph denoted $(Ft(G_\tau), S) = (V, Ft(E))$ such that $e = \{x^+, y^+\} \in Ft(E)$ if there exist a t_s -path, $Q(x^+, y^+) : x^+ e_1 x_1 e_2 x_2 \dots x_k e_k y^+$ in (G_τ, S) .

Proof. According to the directed transitive closure of **K** given in tables 3 and 4 we have: $\mathbf{K} \wedge \mathbf{K} \rightarrow \mathbf{K}$, from where the result. ■

Corollary 8.8. (G_τ, S) is a partial graph of $(Ft(G_\tau), S)$.

Proof. It is obvious from the proposition. ■

Corollary 8.9. $Ft(G_\tau)$ is a negative graph, such that $\forall e \in Ft(E) : \overline{W}(e) = +2$.

Proof. The closure of $Q(x^+, y^+)$ is a negative edge $e = \{x^+, y^+\}$ which is of type **K** and **K** is a negative edge with $\overline{W}(\mathbf{K}) = +2$. ■

Proposition 8.10. Let (G_τ, S) be a directed bi-oriented graph. The transitive closure of (G_τ, S) , is the graph denoted $(Ft(G_\tau), S) = (V, Ft(E))$ such that $e = \{x^-, y^+\} \in Ft(E)$ if there exist a t -path, $Q(x^\alpha, y^\beta) : x^\alpha e_1 x_1 e_2 x_2 \dots x_k e_k y^\beta$ in (G_τ, S) , which is not necessarily a b -path from x^α to y^β , and does not admit edges of type **T**.

Proof. According to the undirected transitive closures of **F**, **U** and **K** given in tables 3 and 4 we have: $\mathbf{U} \wedge \mathbf{F} \rightarrow \mathbf{F}$, $\mathbf{F} \wedge \mathbf{K} \rightarrow \mathbf{F}$, $\mathbf{F} \wedge \mathbf{F} \rightarrow \mathbf{F}$, from where the result. ■

Corollary 8.11. (G_τ, S) is a partial graph of $(Ft(G_\tau), S)$.

Proof. It is obvious from the proposition. ■

Proposition 8.12. Let (G_τ, S) be a directed bi-oriented graph. The transitive closure of (G_τ, S) , is the graph denoted $(Ft(G_\tau), S) = (V, Ft(E))$ such that $e = \{x^+, y^-\} \in Ft(E)$ if there exist a t -path, $Q(x^\alpha, y^\beta) : x^\alpha e_1 x_1 e_2 x_2 \dots x_k e_k y^\beta$ in (G_τ, S) , which is not necessarily a b -path from x^α to y^β and does not admits edges of the type **F**.

Proof. According to the undirected transitive closures of **T**, **U** and **K** given in tables 3 and 4 of we have: $\mathbf{U} \wedge \mathbf{T} \rightarrow \mathbf{T}$, $\mathbf{K} \wedge \mathbf{T} \rightarrow \mathbf{T}$, $\mathbf{T} \wedge \mathbf{T} \rightarrow \mathbf{T}$, from where the result. ■

Corollary 8.13. (G_τ, S) is a partial graph of $(Ft(G_\tau), S)$.

Proof. It is obvious from the proposition. ■

Remarks :

- The directed transitive closure which are given in Proposition 8.1 are identical to the definition of the transitive closure of the bi-oriented graphs given in Definition 3.5.
- The transitive closures which are given in Proposition 8.1 require the existence of a b-path, and they are known as directed positive transitive closures because they are deduced from the directed transitive closures denoted (1) and (2) in section 5.
- The transitive closures which are given in Propositions 8.4 and 8.7 do not require the existence of a b-path, and they are known as directed negative transitive closures because they are deduced from direct transitive closures denoted (3) and (4) in section 5.
- The transitive closures which are given in Propositions 8.10 and 8.12 do not require the existence of a b-path and they are known as hesitation or undirected transitive closures, because they are deduced from the undirected transitive closures denoted (1) - (8) in section 5.

Conclusions

In this paper we propose a first study about the use of signed graphes (more precisely directed bi-oriented graphs) in order to complement the use of logical languages explicitly designed for preference modelling under hesitation. More precisely we show how directed bi-oriented graphs can be used as graphical representation for preference structures based upon the DDT language (a first order four valued logic). In order to complete such new tool we need to introduce new forms of transitive closures (more precisely 8 different forms of transitivity). The result is the establishment of graphical representation tools which enable to use graph theory when preferences are expressed under hesitation and with multiple epistemic states. Two research directions can be followed from these findings. The first concerns the development of ranking and rating procedures exploiting directly the preference structure including hesitation. The second concerns the generalisation of the notion of transitivity as a process for transferring/creating/revising knowledge included in preference statements.

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Annexe A: the complete list of combinations

1	TS	FS	KS	US
TS	ΔS	-	ΔS	-
FS	-	-	-	-
KS	ΔS	-	ΔS	-
US	-	-	-	-

2	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	-	-	-	-
US	-	$\neg\Delta S$	-	$\neg\Delta S$

3	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	-	$\Delta\neg S$	$\Delta\neg S$	-
US	-	-	-	-

4	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	-	-	-
KS	-	-	-	-
US	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$

5	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	-	-	-
KS	ΔS	-	-	ΔS
US	ΔS	-	ΔS	-

6	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	-	$\neg\Delta S$	-	$\neg\Delta S$
US	-	$\neg\Delta S$	$\neg\Delta S$	-

7	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	-	$\Delta\neg S$	-	$\Delta\neg S$
US	-	$\Delta\neg S$	$\Delta\neg S$	-

8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	-	-	-
KS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

1234567	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	KS	FS $\Delta\neg S$
US	TS	FS	$\Delta\neg S$	US

1234568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	$\neg\Delta S$
KS	TS	FS	KS	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	US

1234578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS
KS	TS	$\Delta\neg S$	KS	ΔS
US	TS	FS	ΔS	US

1234678	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	TS	FS	KS	$\neg\Delta S$
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	US

1235678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	KS	-
US	TS	FS	-	$\neg\Delta S$

1245678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	ΔS	-
US	TS	FS	-	US

1345678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	KS	-
US	TS	FS	-	$\neg\Delta\neg S$

2345678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	$\Delta\neg S$	-
US	TS	FS	-	US

12	TS	FS	KS	US
TS	ΔS	-	ΔS	-
FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	-	ΔS	-
US	-	$\neg\Delta S$	-	$\neg\Delta S$

13	TS	FS	KS	US
TS	ΔS	-	ΔS	-
FS	-	$\neg\Delta S$	$\neg\Delta S$	-
KS	ΔS	$\neg\Delta S$	KS -	-
US	-	-	-	-

14	TS	FS	KS	US
TS	-	-	ΔS	$\neg\Delta\neg S$
FS	-	-	-	-
KS	ΔS	-	ΔS	-
US	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$

15	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	-	-	-
KS	ΔS	-	ΔS	ΔS
US	ΔS	-	ΔS	-

16	TS	FS	KS	US
TS	ΔS	-	ΔS	-
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
US	-	$\neg\Delta S$	$\neg\Delta S$	-

17	TS	FS	KS	US
TS	ΔS	-	ΔS	-
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	ΔS	$\Delta\neg S$	ΔS	$\Delta\neg S$
US	-	$\Delta\neg S$	$\Delta\neg S$	-

18	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	-	-	-
KS	TS	-	ΔS	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-

78	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	-	-
US	$\neg\Delta\neg S$	-	$\Delta\neg S$	-

23	TS	FS	KS	US
TS	-	-	-	-
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	-	$\Delta\neg S$	$\Delta\neg S$	-
US	-	$\neg\Delta S$	-	$\neg\Delta S$

24	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	-	-	-	-
US	$\neg\Delta\neg S$	$\neg\Delta S$	-	US

25	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	-	-	ΔS
US	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$

26	TS	FS	KS	US
TS	-	-	-	-
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	-	$\neg\Delta S$	-	$\neg\Delta S$
US	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$

27	TS	FS	KS	US
TS	-	-	-	-
FS	-	FS	$\Delta\neg S$	FS
KS	-	$\Delta\neg S$	-	$\Delta\neg S$
US	-	FS	$\Delta\neg S$	$\Delta\neg S$

28	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

67	TS	FS	KS	US
TS	-	-	-	-
FS	-	FS	FS	FS
KS	-	FS	-	FS
US	-	FS	FS	-

68	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\neg\Delta S$	-	US
US	$\neg\Delta\neg S$	$\neg\Delta S$	US	-

34	TS	FS	KS	US	35	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	$\Delta\neg S$	$\Delta\neg S$	-	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	-	$\Delta\neg S$	ΔS	-	KS	ΔS	$\Delta\neg S$	$\Delta\neg S$	-
US	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	US	ΔS	-	-	-

3 6	TS	FS	KS	US	3 7	TS	FS	KS	US
TS	-	-	-	-	TS	-	-	-	-
FS	-	FS	FS	$\neg\Delta S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	-	FS	$\Delta\neg S$	$\neg\Delta S$	KS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
US	-	$\neg\Delta S$	$\neg\Delta S$	-	US	-	$\Delta\neg S$	$\Delta\neg S$	-

3 8	TS	FS	KS	US	56	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	$\Delta\neg S$	$\Delta\neg S$	-	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$	KS	ΔS	$\neg\Delta S$	-	-
US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-	US	ΔS	$\neg\Delta S$	-	-

5 7	TS	FS	KS	US	5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	-	-	-
KS	ΔS	$\Delta\neg S$	-	KS	KS	TS	-	-	TS
US	ΔS	$\Delta\neg S$	KS	-	US	TS	-	TS	-

45	TS	FS	KS	US	46	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	-	-	-	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	-	-	ΔS	KS	-	$\neg\Delta S$	-	$\neg\Delta S$
US	TS	-	ΔS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$

4 7	TS	FS	KS	US	4 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	-	-	-
KS	-	$\Delta\neg S$	-	$\Delta\neg S$	KS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$

1 45	TS	FS	KS	US	1 46	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	ΔS	$\neg\Delta\neg S$
FS	-	-	-	-	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	-	ΔS	ΔS	KS	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
US	TS	-	ΔS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$

1 4 7	TS	FS	KS	US	1 4 8	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	-	-	-
KS	ΔS	$\Delta\neg S$	ΔS	$\Delta\neg S$	KS	TS	-	ΔS	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$

123	TS	FS	KS	US	124	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	TS	-	ΔS	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	KS	-	KS	ΔS	-	ΔS	-
US	-	$\neg\Delta S$	-	$\neg\Delta S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	-	US
12 5	TS	FS	KS	US	12 6	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	ΔS	-	ΔS	-
FS	-	$\neg\Delta S$	-	$\neg\Delta S$	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	-	ΔS	ΔS	KS	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
US	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$	US	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
12 7	TS	FS	KS	US	12 8	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	ΔS	$\Delta\neg S$	KS	TS	-	ΔS	$\neg\Delta\neg S$
US	-	FS	$\Delta\neg S$	$\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$
1 67	TS	FS	KS	US	1 68	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	FS	ΔS	FS	KS	TS	$\neg\Delta S$	ΔS	US
US	-	FS	FS	-	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	-
134	TS	FS	KS	US	135	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	$\Delta\neg S$	$\Delta\neg S$	-	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	ΔS	$\Delta\neg S$	KS	ΔS	KS	ΔS	$\Delta\neg S$	KS	ΔS
US	$\neg\Delta\neg S$	-	ΔS	$\neg\Delta\neg S$	US	ΔS	-	ΔS	-
13 6	TS	FS	KS	US	13 7	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	ΔS	-	ΔS	-
FS	-	FS	FS	$\neg\Delta S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	ΔS	FS	KS	$\neg\Delta S$	KS	ΔS	$\Delta\neg S$	KS	$\Delta\neg S$
US	-	$\neg\Delta S$	$\neg\Delta S$	-	US	-	$\Delta\neg S$	$\Delta\neg S$	-
13 8	TS	FS	KS	US	1 56	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	$\Delta\neg S$	$\Delta\neg S$	-	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\Delta\neg S$	KS	$\neg\Delta\neg S$	KS	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$
US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	-	US	ΔS	$\neg\Delta S$	$\neg\Delta S$	-
1 57	TS	FS	KS	US	1 5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	-	-	-
KS	ΔS	$\Delta\neg S$	ΔS	KS	KS	TS	-	ΔS	TS
US	ΔS	$\Delta\neg S$	KS	-	US	TS	-	TS	-

234	TS	FS	KS	US	23 5	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	-	$\Delta\neg S$	ΔS	-	KS	ΔS	$\Delta\neg S$	$\Delta\neg S$	ΔS
US	$\neg\Delta\neg S$	$\neg\Delta S$	-	US	US	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$

23 6	TS	FS	KS	US	23 7	TS	FS	KS	US
TS	-	-	-	-	TS	-	-	-	-
FS	-	FS	FS	$\neg\Delta S$	FS	-	FS	$\Delta\neg S$	$\Delta\neg S$
KS	-	FS	$\Delta\neg S$	$\neg\Delta S$	KS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
US	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$	US	-	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta S$

23 8	TS	FS	KS	US	2 56	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$	KS	ΔS	$\neg\Delta S$	-	-
US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$	US	ΔS	$\neg\Delta S$	-	$\neg\Delta S$

2 57	TS	FS	KS	US	2 5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	-	KS	KS	TS	-	-	TS
US	ΔS	FS	KS	$\neg\Delta S$	US	TS	$\neg\Delta S$	TS	$\neg\Delta S$

2 45	TS	FS	KS	US	2 4 6	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	$\neg\Delta S$	-	$\neg\Delta S$	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	Δ	-	-	ΔS	KS	-	$\neg\Delta S$	-	$\neg\Delta S$
US	TS	$\neg\Delta S$	ΔS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	US

2 4 7	TS	FS	KS	US	2 4 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	-	$\Delta\neg S$	-	$\Delta\neg S$	KS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	US

1 78	TS	FS	KS	US	2 78	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	FS	$\Delta\neg S$	FS
KS	TS	$\Delta\neg S$	ΔS	-	KS	$\neg\Delta\neg S$	$\Delta\neg S$	-	-
US	$\neg\Delta\neg S$	$\Delta\neg S$	-	-	US	$\neg\Delta\neg S$	FS	-	$\neg\Delta S$

2 67	TS	FS	KS	US	2 6 8	TS	FS	KS	US
TS	-	-	-	-	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	-	FS	-	FS	KS	$\neg\Delta\neg S$	$\neg\Delta S$	-	US
US	-	FS	FS	$\neg\Delta S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta S$

345	TS	FS	KS	US	34 6	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	-	FS	-	FS	FS	$\neg\Delta S$
KS	Δ	$\Delta\neg S$	$\Delta\neg S$	ΔS	KS	-	FS	$\Delta\neg S$	$\neg\Delta S$
US	TS	-	ΔS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$
34 7	TS	FS	KS	US	34 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
3 67	TS	FS	KS	US	3 6 8	TS	FS	KS	US
TS	-	-	-	-	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	-	FS	$\Delta\neg S$	FS	KS	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US
US	-	FS	FS	-	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	-
4 67	TS	FS	KS	US	4 6 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	-	FS	-	FS	KS	$\neg\Delta\neg S$	$\neg\Delta S$	-	US
US	$\neg\Delta\neg S$	FS	FS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta\neg S$
3 78	TS	FS	KS	US	3 56	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	FS	FS	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-	KS	ΔS	FS	$\Delta\neg S$	-
US	$\neg\Delta\neg S$	$\Delta\neg S$	-	-	US	ΔS	$\neg\Delta S$	-	-
3 5 7	TS	FS	KS	US	3 5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	ΔS	$\Delta\neg S$	$\Delta\neg S$	KS	KS	TS	$\Delta\neg S$	$\Delta\neg S$	TS
US	ΔS	$\Delta\neg S$	KS	-	US	TS	-	TS	-
4 78	TS	FS	KS	US	456	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	-	-	KS	ΔS	$\neg\Delta S$	-	-
US	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$	US	TS	$\neg\Delta S$	-	$\neg\Delta\neg S$
45 7	TS	FS	KS	US	45 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	-	-	-
KS	ΔS	$\Delta\neg S$	-	$\Delta\neg S$	KS	TS	-	-	TS
US	TS	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$	US	TS	-	TS	$\neg\Delta\neg S$

1234	TS	FS	KS	US	123 5	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	KS	-	KS	ΔS	$\Delta\neg S$	KS	ΔS
US	$\neg\Delta\neg S$	$\neg\Delta S$	-	US	US	ΔS	$\neg\Delta S$	ΔS	$\neg\Delta S$

123 6	TS	FS	KS	US	123 7	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	ΔS	-	ΔS	-
FS	-	FS	FS	$\neg\Delta S$	FS	-	FS	$\Delta\neg S$	$\Delta\neg S$
KS	ΔS	FS	KS	$\neg\Delta S$	KS	ΔS	$\Delta\neg S$	KS	$\Delta\neg S$
US	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$	US	-	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta S$

123 8	TS	FS	KS	US	12 56	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\Delta\neg S$	KS	$\neg\Delta\neg S$	KS	ΔS	$\neg\Delta S$	ΔS	-
US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$	US	ΔS	$\neg\Delta S$	-	$\neg\Delta S$

12 57	TS	FS	KS	US	12 5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	ΔS	KS	KS	TS	-	ΔS	TS
US	ΔS	FS	KS	$\neg\Delta S$	US	TS	$\neg\Delta S$	TS	$\neg\Delta S$

12 45	TS	FS	KS	US	12 4 6	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	$\neg\Delta S$	-	$\neg\Delta S$	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	Δ	-	ΔS	ΔS	KS	-	$\neg\Delta S$	ΔS	$\neg\Delta S$
US	TS	$\neg\Delta S$	ΔS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	US

12 4 7	TS	FS	KS	US	12 4 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	-	$\Delta\neg S$	ΔS	$\Delta\neg S$	KS	TS	-	ΔS	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	US

678	TS	FS	KS	US	12 78	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	$\Delta\neg S$	FS
KS	$\neg\Delta\neg S$	FS	-	$\neg\Delta S$	KS	TS	$\Delta\neg S$	ΔS	-
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	-	US	$\neg\Delta\neg S$	FS	-	$\neg\Delta S$

12 67	TS	FS	KS	US	12 6 8	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	FS	ΔS	FS	KS	TS	$\neg\Delta S$	ΔS	US
US	-	FS	FS	$\neg\Delta S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta S$

1 3 4 5	TS	FS	KS	US	1 3 4 6	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	ΔS	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	-	FS	-	FS	FS	$\neg\Delta S$
KS	Δ	$\Delta\neg S$	KS	ΔS	KS	ΔS	FS	KS	$\neg\Delta S$
US	TS	-	ΔS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta\neg S$
1 3 4 7	TS	FS	KS	US	1 3 4 8	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	ΔS	$\Delta\neg S$	KS	$\Delta\neg S$	KS	TS	$\Delta\neg S$	KS	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
1 3 6 7	TS	FS	KS	US	1 3 6 8	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	ΔS	FS	KS	FS	KS	TS	FS	KS	US
US	-	FS	FS	-	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	-
1 4 6 7	TS	FS	KS	US	1 4 6 8	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	FS	ΔS	FS	KS	TS	$\neg\Delta S$	ΔS	US
US	$\neg\Delta\neg S$	FS	FS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta\neg S$
1 3 7 8	TS	FS	KS	US	1 3 5 6	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	FS	FS	$\neg\Delta S$
KS	TS	$\Delta\neg S$	KS	-	KS	ΔS	FS	KS	-
US	$\neg\Delta\neg S$	$\Delta\neg S$	-	-	US	ΔS	$\neg\Delta S$	-	-
1 3 5 7	TS	FS	KS	US	1 3 5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	ΔS	$\Delta\neg S$	KS	KS	KS	TS	$\Delta\neg S$	KS	TS
US	ΔS	$\Delta\neg S$	KS	-	US	TS	-	TS	-
1 4 7 8	TS	FS	KS	US	1 4 5 6	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\Delta\neg S$	ΔS	-	KS	ΔS	$\neg\Delta S$	ΔS	-
US	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$	US	TS	$\neg\Delta S$	-	$\neg\Delta\neg S$
1 4 5 7	TS	FS	KS	US	1 4 5 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	-	-	-
KS	ΔS	$\Delta\neg S$	ΔS	KS	KS	TS	-	ΔS	TS
US	TS	$\Delta\neg S$	KS	$\neg\Delta\neg S$	US	TS	-	TS	$\neg\Delta\neg S$

2345	TS	FS	KS	US	234 6	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	-	FS	-	FS	FS	$\neg\Delta S$
KS	Δ	$\Delta\neg S$	$\Delta\neg S$	ΔS	KS	-	FS	$\Delta\neg S$	$\neg\Delta S$
US	TS	-	ΔS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	US
234 7	TS	FS	KS	US	234 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	US
23 67	TS	FS	KS	US	23 6 8	TS	FS	KS	US
TS	-	-	-	-	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	-	FS	$\Delta\neg S$	FS	KS	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US
US	-	FS	FS	$\neg\Delta S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta S$
2 4 67	TS	FS	KS	US	2 4 6 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	-	FS	-	FS	KS	$\neg\Delta\neg S$	$\neg\Delta S$	-	US
US	$\neg\Delta\neg S$	FS	FS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	US
23 78	TS	FS	KS	US	23 56	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	FS	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-	KS	ΔS	FS	$\Delta\neg S$	-
US	$\neg\Delta\neg S$	FS	-	-	US	ΔS	$\neg\Delta S$	-	$\neg\Delta S$
23 5 7	TS	FS	KS	US	23 5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	$\Delta\neg S$	KS	KS	TS	$\Delta\neg S$	$\Delta\neg S$	TS
US	ΔS	FS	KS	$\neg\Delta S$	US	TS	$\neg\Delta S$	TS	$\neg\Delta S$
2 4 78	TS	FS	KS	US	2 456	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	-	-	KS	ΔS	$\neg\Delta S$	-	-
US	$\neg\Delta\neg S$	FS	-	US	US	TS	$\neg\Delta S$	-	US
2 45 7	TS	FS	KS	US	2 45 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	-	KS	KS	TS	-	-	TS
US	TS	FS	KS	US	US	TS	$\neg\Delta S$	TS	US

567	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	FS	FS	FS
KS	ΔS	FS	-	FS
US	ΔS	FS	FS	-

568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\neg\Delta S$	-	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	-

578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	TS	$\Delta\neg S$	-	ΔS
US	TS	$\Delta\neg S$	ΔS	-

5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	-	-
US	TS	FS	-	-

1 567	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	FS	FS	FS
KS	ΔS	FS	ΔS	-
US	ΔS	FS	-	-

1 568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\neg\Delta S$	ΔS	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	-

1 578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	TS	$\Delta\neg S$	ΔS	ΔS
US	TS	$\Delta\neg S$	ΔS	-

1 678	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	TS	FS	ΔS	$\neg\Delta S$
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	-

2 567	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	FS	FS	FS
KS	ΔS	FS	-	-
US	ΔS	FS	-	$\neg\Delta S$

2 568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\neg\Delta S$	-	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

2 578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS
KS	TS	$\Delta\neg S$	-	ΔS
US	TS	FS	ΔS	$\neg\Delta S$

2 678	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	$\neg\Delta\neg S$	FS	-	$\neg\Delta S$
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	$\neg\Delta S$

3 567	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	FS	FS	FS
KS	ΔS	FS	$\Delta\neg S$	-
US	ΔS	FS	-	-

3 568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	$\neg\Delta S$
KS	TS	FS	$\Delta\neg S$	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	-

3 578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	TS	$\Delta\neg S$	$\Delta\neg S$	ΔS
US	TS	$\Delta\neg S$	ΔS	-

3 678	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	$\neg\Delta\neg S$	FS	$\Delta\neg S$	$\neg\Delta S$
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	-

12345	TS	FS	KS	US	1234 6	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	ΔS	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	$\neg\Delta S$	FS	-	FS	FS	$\neg\Delta S$
KS	Δ	$\Delta\neg S$	KS	ΔS	KS	ΔS	FS	KS	$\neg\Delta S$
US	TS	$\neg\Delta S$	ΔS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta S$	US
1234 7	TS	FS	KS	US	1234 8	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	KS	$\Delta\neg S$	KS	TS	$\Delta\neg S$	KS	$\neg\Delta\neg S$
US	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	$\neg\Delta\neg S$	US
123 67	TS	FS	KS	US	123 6 8	TS	FS	KS	US
TS	ΔS	-	ΔS	-	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	ΔS	FS	KS	FS	KS	TS	FS	KS	US
US	-	FS	FS	$\neg\Delta S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta S$
12 4 67	TS	FS	KS	US	12 4 6 8	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	ΔS	FS	ΔS	FS	KS	TS	$\neg\Delta S$	ΔS	US
US	$\neg\Delta\neg S$	FS	FS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	US
123 78	TS	FS	KS	US	123 56	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	ΔS	-	ΔS	ΔS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	FS	$\neg\Delta S$
KS	TS	$\Delta\neg S$	KS	-	KS	ΔS	FS	KS	-
US	$\neg\Delta\neg S$	FS	-	$\neg\Delta S$	US	ΔS	$\neg\Delta S$	-	$\neg\Delta S$
123 5 7	TS	FS	KS	US	123 5 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	KS	$\Delta\neg S KS$	KS	TS	$\Delta\neg S$	KS	TS
US	ΔS	FS	KS	$\neg\Delta S$	US	TS	$\neg\Delta S$	TS	$\neg\Delta S$
12 4 78	TS	FS	KS	US	12 456	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	FS	$\neg\Delta S$
KS	TS	$\Delta\neg S$	ΔS	-	KS	ΔS	FS	KS	-
US	$\neg\Delta\neg S$	FS	-	US	US	TS	$\neg\Delta S$	-	US
12 45 7	TS	FS	KS	US	12 45 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	$\neg\Delta S$	-	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	ΔS	KS	KS	TS	-	ΔS	TS
US	TS	FS	KS	US	US	TS	$\neg\Delta S$	TS	US

4567	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	-	$\Delta\neg S$
US	TS	FS	$\Delta\neg S$	$\neg\Delta\neg S$

4568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\neg\Delta S$	-	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta\neg S$

4578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	TS	$\Delta\neg S$	-	ΔS
US	TS	$\Delta\neg S$	ΔS	$\neg\Delta\neg S$

4678	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	$\neg\Delta\neg S$	FS	-	$\neg\Delta S$
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	$\neg\Delta\neg S$

12567	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	FS	FS	FS
KS	ΔS	FS	ΔS	$\Delta\neg S$
US	ΔS	FS	$\Delta\neg S$	$\neg\Delta S$

12568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\neg\Delta S$	ΔS	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

12578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS
KS	TS	$\Delta\neg S$	ΔS	ΔS
US	TS	FS	ΔS	$\neg\Delta S$

12678	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	TS	FS	ΔS	$\neg\Delta S$
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	$\neg\Delta S$

13567	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	FS	FS	FS
KS	ΔS	FS	KS	$\Delta\neg S$
US	ΔS	FS	$\Delta\neg S$	-

13568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	$\neg\Delta S$
KS	TS	FS	KS	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	-

13578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	TS	$\Delta\neg S$	KS	ΔS
US	TS	$\Delta\neg S$	ΔS	-

13678	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	TS	FS	KS	$\neg\Delta S$
US	$\neg\Delta\neg S$	$\neg\Delta S$	FS	-

14567	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	ΔS	$\Delta\neg S$
US	TS	FS	$\Delta\neg S$	$\neg\Delta\neg S$

14568	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg\Delta S$	$\neg\Delta S$	$\neg\Delta S$
KS	TS	$\neg\Delta S$	ΔS	$\neg\Delta\neg S$
US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta\neg S$

14578	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$
KS	TS	$\Delta\neg S$	ΔS	ΔS
US	TS	$\Delta\neg S$	ΔS	$\neg\Delta\neg S$

14678	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS
KS	TS	FS	ΔS	$\neg\Delta S$
US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	$\neg\Delta\neg S$

34 78	TS	FS	KS	US	3456	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	FS	FS	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-	KS	ΔS	FS	$\Delta\neg S$	-
US	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$	US	TS	$\neg\Delta S$	-	$\neg\Delta\neg S$

345 7	TS	FS	KS	US	345 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	ΔS	$\Delta\neg S$	$\Delta\neg S$	KS	KS	TS	$\Delta\neg S$	$\Delta\neg S$	TS
US	TS	$\Delta\neg S$	KS	$\neg\Delta\neg S$	US	TS	-	TS	$\neg\Delta\neg S$

34 67	TS	FS	KS	US	34 6 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	-	FS	$\Delta\neg S$	FS	KS	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US
US	$\neg\Delta\neg S$	FS	FS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta\neg S$

1 34 78	TS	FS	KS	US	1 3456	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	FS	FS	$\neg\Delta S$
KS	TS	$\Delta\neg S$	KS	-	KS	ΔS	FS	KS	-
US	$\neg\Delta\neg S$	$\Delta\neg S$	-	$\neg\Delta\neg S$	US	TS	$\neg\Delta S$	-	$\neg\Delta\neg S$

1 345 7	TS	FS	KS	US	1 345 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	$\Delta\neg S$	$\Delta\neg S$	-
KS	ΔS	$\Delta\neg S$	KS	KS	KS	TS	$\Delta\neg S$	KS	TS
US	TS	$\Delta\neg S$	KS	$\neg\Delta\neg S$	US	TS	-	TS	$\neg\Delta\neg S$

1 34 67	TS	FS	KS	US	1 34 6 8	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	ΔS	FS	KS	FS	KS	TS	FS	KS	US
US	$\neg\Delta\neg S$	FS	FS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	$\neg\Delta\neg S$

1 34567	TS	FS	KS	US	1 3456 8	TS	FS	KS	US
TS	TS	-	TS	TS	TS	TS	-	TS	TS
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	TS	FS	KS	$\Delta\neg S$	KS	TS	FS	KS	$\neg\Delta\neg S$
US	TS	FS	$\Delta\neg S$	$\neg\Delta\neg S$	US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta\neg S$

1 345 78	TS	FS	KS	US	1 34 678	TS	FS	KS	US
TS	TS	-	TS	TS	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	$\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	FS	-	FS	FS	FS
KS	TS	$\Delta\neg S$	KS	ΔS	KS	TS	FS	ΔS	$\neg\Delta S$
US	TS	$\Delta\neg S$	ΔS	$\neg\Delta\neg S$	US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	$\neg\Delta\neg S$

23 567	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS
FS	-	FS	FS	FS
KS	ΔS	FS	$\Delta \neg S$	$\Delta \neg S$
US	ΔS	FS	$\Delta \neg S$	$\neg \Delta S$

23 56 8	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	$\neg \Delta S$
KS	TS	FS	$\Delta \neg S$	$\neg \Delta \neg S$
US	TS	$\neg \Delta S$	$\neg \Delta \neg S$	$\neg \Delta S$

23 5 78	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	$\Delta \neg S$	FS
KS	TS	$\Delta \neg S$	$\Delta \neg S$	ΔS
US	TS	FS	ΔS	$\neg \Delta S$

23 678	TS	FS	KS	US
TS	$\neg \Delta \neg S$	-	$\neg \Delta \neg S$	$\neg \Delta \neg S$
FS	-	FS	FS	FS
KS	$\neg \Delta \neg S$	FS	$\Delta \neg S$	$\neg \Delta S$
US	$\neg \Delta \neg S$	FS	$\neg \Delta S$	$\neg \Delta S$

2 4567	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	-	$\Delta \neg S$
US	TS	FS	$\Delta \neg S$	US

2 456 8	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg \Delta S$	$\neg \Delta S$	$\neg \Delta S$
KS	TS	$\neg \Delta S$	-	$\neg \Delta \neg S$
US	TS	$\neg \Delta S$	$\neg \Delta \neg S$	US

2 45 78	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	$\Delta \neg S$	FS
KS	TS	$\Delta \neg S$	-	ΔS
US	TS	FS	ΔS	US

2 4 678	TS	FS	KS	US
TS	$\neg \Delta \neg S$	-	$\neg \Delta \neg S$	$\neg \Delta \neg S$
FS	-	FS	FS	FS
KS	$\neg \Delta \neg S$	FS	-	$\neg \Delta S$
US	$\neg \Delta \neg S$	FS	$\neg \Delta S$	US

34567	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	$\Delta \neg S$	$\Delta \neg S$
US	TS	FS	$\Delta \neg S$	$\neg \Delta \neg S$

3456 8	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	$\neg \Delta S$
KS	TS	FS	$\Delta \neg S$	$\neg \Delta S$
US	TS	$\neg \Delta S$	$\neg \Delta S$	$\neg \Delta \neg S$

345 78	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\Delta \neg S$	$\Delta \neg S$	$\Delta \neg S$
KS	TS	$\Delta \neg S$	$\Delta \neg S$	ΔS
US	TS	$\Delta \neg S$	ΔS	$\neg \Delta \neg S$

34 678	TS	FS	KS	US
TS	$\neg \Delta \neg S$	-	$\neg \Delta \neg S$	$\neg \Delta \neg S$
FS	-	FS	FS	FS
KS	$\neg \Delta \neg S$	FS	ΔS	$\neg \Delta S$
US	$\neg \Delta \neg S$	FS	$\neg \Delta S$	$\neg \Delta \neg S$

1 5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	ΔS	-
US	TS	FS	-	-

2 5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	-	-
US	TS	FS	-	$\neg \Delta S$

3 5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	$\neg \Delta S$	-
US	TS	FS	-	-

45678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	-	-
US	TS	FS	-	$\neg \Delta \neg S$

234 78	TS	FS	KS	US	23456	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	FS	$\neg\Delta S$
KS	$\neg\Delta\neg S$	$\Delta\neg S$	$\Delta\neg S$	-	KS	ΔS	FS	$\Delta\neg S$	-
US	$\neg\Delta\neg S$	FS	-	US	US	TS	$\neg\Delta S$	-	US

2345 7	TS	FS	KS	US	2345 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	$\Delta\neg S$	KS	KS	TS	$\Delta\neg S$	$\Delta\neg S$	TS
US	TS	FS	KS	US	US	TS	$\neg\Delta S$	TS	US

234 67	TS	FS	KS	US	234 6 8	TS	FS	KS	US
TS	$\neg\Delta\neg S$	-	-	$\neg\Delta\neg S$	TS	$\neg\Delta\neg S$	-	$\neg\Delta\neg S$	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	-	FS	$\Delta\neg S$	FS	KS	$\neg\Delta\neg S$	FS	$\Delta\neg S$	US
US	$\neg\Delta\neg S$	FS	FS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	US

1234 78	TS	FS	KS	US	123456	TS	FS	KS	US
TS	TS	-	TS	$\neg\Delta\neg S$	TS	TS	-	ΔS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	FS	$\neg\Delta S$
KS	TS	$\Delta\neg S$	KS	-	KS	ΔS	FS	KS	-
US	$\neg\Delta\neg S$	FS	-	US	US	TS	$\neg\Delta S$	-	US

12345 7	TS	FS	KS	US	12345 8	TS	FS	KS	US
TS	TS	-	ΔS	TS	TS	TS	-	TS	TS
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	$\Delta\neg S$	$\neg\Delta S$
KS	ΔS	$\Delta\neg S$	KS	KS	KS	TS	$\Delta\neg S$	KS	TS
US	TS	FS	KS	US	US	TS	$\neg\Delta S$	TS	US

1234 67	TS	FS	KS	US	1234 6 8	TS	FS	KS	US
TS	TS	-	ΔS	$\neg\Delta\neg S$	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	ΔS	FS	KS	FS	KS	TS	FS	KS	US
US	$\neg\Delta\neg S$	FS	FS	US	US	$\neg\Delta\neg S$	$\neg\Delta S$	US	US

123 567	TS	FS	KS	US	123 56 8	TS	FS	KS	US
TS	ΔS	-	ΔS	ΔS	TS	TS	-	TS	TS
FS	-	FS	FS	FS	FS	-	FS	FS	$\neg\Delta S$
KS	ΔS	FS	KS	$\Delta\neg S$	KS	TS	FS	KS	$\neg\Delta\neg S$
US	ΔS	FS	$\Delta\neg S$	$\neg\Delta S$	US	TS	$\neg\Delta S$	$\neg\Delta\neg S$	$\neg\Delta S$

123 5 78	TS	FS	KS	US	123 678	TS	FS	KS	US
TS	TS	-	TS	TS	TS	TS	-	TS	$\neg\Delta\neg S$
FS	-	FS	$\Delta\neg S$	FS	FS	-	FS	FS	FS
KS	TS	$\Delta\neg S$	KS	ΔS	KS	TS	FS	KS	$\neg\Delta S$
US	TS	FS	ΔS	$\neg\Delta S$	US	$\neg\Delta\neg S$	FS	$\neg\Delta S$	$\neg\Delta S$

12 4567	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	ΔS	$\Delta \neg S$
US	TS	FS	$\Delta \neg S$	US

12 456 8	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	$\neg \Delta S$	$\neg \Delta S$	$\neg \Delta S$
KS	TS	$\neg \Delta S$	ΔS	$\neg \Delta \neg S$
US	TS	$\neg \Delta S$	$\neg \Delta \neg S$	US

12 45 78	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	$\Delta \neg S$	FS
KS	TS	$\Delta \neg S$	ΔS	ΔS
US	TS	FS	ΔS	US

12 4 678	TS	FS	KS	US
TS	TS	-	TS	$\neg \Delta \neg S$
FS	-	FS	FS	FS
KS	TS	FS	ΔS	$\neg \Delta S$
US	$\neg \Delta \neg S$	FS	$\neg \Delta S$	US

234567	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	$\Delta \neg S$	$\Delta \neg S$
US	TS	FS	$\Delta \neg S$	US

23456 8	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	$\neg \Delta S$
KS	TS	FS	$\Delta \neg S$	$\neg \Delta \neg S$
US	TS	$\neg \Delta S$	$\neg \Delta \neg S$	US

2345 78	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	$\Delta \neg S$	FS
KS	TS	$\Delta \neg S$	$\Delta \neg S$	ΔS
US	TS	FS	ΔS	US

234 678	TS	FS	KS	US
TS	$\neg \Delta \neg S$	-	$\neg \Delta \neg S$	$\neg \Delta \neg S$
FS	-	FS	FS	FS
KS	$\neg \Delta \neg S$	FS	ΔS	$\neg \Delta S$
US	$\neg \Delta \neg S$	FS	$\neg \Delta S$	US

12 5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	ΔS	-
US	TS	FS	-	$\neg \Delta S$

1 3 5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	KS	-
US	TS	FS	-	-

345678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	$\neg \Delta S$	-
US	TS	FS	-	$\neg \Delta \neg S$

1 45678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	ΔS	-
US	TS	FS	-	$\neg \Delta \neg S$

23 5678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	$\Delta \neg S$	-
US	TS	FS	-	$\neg \Delta S$

2 45678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	-	-
US	TS	FS	-	US

12345678	TS	FS	KS	US
TS	TS	-	TS	TS
FS	-	FS	FS	FS
KS	TS	FS	KS	-
US	TS	FS	-	US