Rating with Multiple Criteria

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Outline

What is a rating problem?

Primitive

The primitive is a binary relation on A: $\succeq \subseteq A \times P \cup P \times A$
to be read "at least as good as".
P being the set of external "norms" characterising the ordered classes $C_1 \rhd \cdots \rhd C_n$

Result

The result is to assign each element of A in a C_j such that:
$x \in C_j \;\Leftrightarrow\; x \succeq' p_j, p_{j+1}, \cdots p_n$ and $p_1 \cdots p_{j-1} \succeq' x$

Discussion 1

Constraint Satisfaction

If $\forall x, y \in A \cup P \;\; x \succeq y \;\Leftrightarrow\; f(x) \geq f(y)$.
Then $x \in C_j \;\Leftrightarrow\; f(p_{j-1}) \geq f(x) \geq f(p_j)$.
This is a Constraint Satisfaction Problem.

Why is $\succeq'$ different from $\succeq$?

Generally speaking $\succeq$ is not an ordering relation since preferences can be partial and or inconsistent. If we have to proceed with some operational procedure we need to transform $\succeq$ to an ordering relation $\succeq'$.

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What do we have?

- a set A of items to be rated;
- each element of A is described against a set of attributes D;
- a set C of classes (the rates);
- a set P of “profiles” associated to the classes;
- each element of P is described against the same set of attributes D;
- preference statements among elements of each attribute;
- preference statements among coalitions of attributes.

To pay attention

- classes are linearly ordered: $C_1 \rhd \cdots \rhd C_n$;
- "profiles" represent the minimal (maximal) requirements for an object to be rated in a certain class;
- if we have n classes we have $n - 1$ profiles (representing the boundary among classes);
- "profiles" are linearly ordered;
- pairwise comparisons occur between members of A and members of P;

First step 1

Establish the outranking relation on the basis of the following rule:

$$s(x,y) \quad \Leftrightarrow \quad C(x,y) \textbf{ and not } D(x,y)$$

where

$$\forall x \in A,\ y \in \mathcal{P}:\ C(x,y) \quad \Leftrightarrow \quad \sum_{j \in G^{\pm}} w_j \geq c \ \textbf{ and } \ (\sum_{j \in G^{+}} w_j \geq \sum_{j \in G^{-}} w_j)$$

First step 2

$$\forall y \in A,\ x \in \mathcal{P}:\ C(x,y) \quad \Leftrightarrow$$
$$\Big(\sum_{j \in G^{\pm}} w_j \geq c \ \textbf{ and } \sum_{j \in G^{+}} w_j \geq \sum_{j \in G^{-}} w_j\Big) \ \textbf{ or } \ \Big(\sum_{j \in G^{+}} w_j > \sum_{j \in G^{-}} w_j\Big)$$

First step 3

$$\forall (x,y) \in (A \times \mathcal{P}) \cup (\mathcal{P} \times A) : \textbf{ not } D(x,y) \quad \Leftrightarrow$$
$$\sum_{j \in G^-} w_j \leq d \textbf{ and } \forall g_j \textbf{ not } v_j(x,y)$$

First step 4

where

- $G^{+} = \{g_j \in G : P_j(x, y)\}$
- $G^{-} = \{g_j \in G : P_j(y, x)\}$
- $G^{=} = \{g_j \in G : I_j(x, y)\}$
- $G^{\pm} = G^{+} \cup G^{=}$
- c: the concordance threshold $c \in [0.5, 1]$
- d: the discordance threshold $d \in [0, 1]$
- $v_j(x, y)$: veto, expressed on criterion g_j, of y on x

Second step

When the relation S is established, assign any element a_i on the basis of the following rules.

2.1 pessimistic assignment
- a_i is iteratively compared with $p_t \cdots p_1$,
- as soon as $s(a_i, p_h)$ is established, assign a_i to category c_h.

2.2 optimistic assignment
- a_i is iteratively compared with $p_1 \cdots p_t$,
- as soon as is established $s(p_h, a_i) \ \wedge \ \neg \ s(a_i, p_h)$ then assign a_i to category c_{h-1}.

In other terms ...

Summarising

The pessimistic procedure finds the profile for which the element is not the worst. The optimistic procedure finds the profile against which the element is surely the worse. If the optimistic and pessimistic assignments coincide, then no uncertainty exists for the assignment. Otherwise, an uncertainty exists and should be considered by the user.

Divina Commedia

	A1	A2	A3	A4	A5	A6
Criterion 1	550	950	300	650	700	480
Criterion 2	9	4	7	2	5	11
Criterion 3	48	35	20	80	52	77

P1 = (500, 5, 35)
P2 = (800, 9, 70)