Multiple Criteria Decision Analysis

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Outline

1. What is the problem?
2. The Borda path
   - How Better?
   - Comparing apples to peaches
   - Example
3. The Condorcet path
4. Conclusions

Example

Consider the following evaluation table concerning four candidates (A,B,C and D) assessed against four criteria H1,H2,H3 and H4.

	H1	H2	H3	H4
A	7	5	9	6
B	8	4	7	8
C	5	8	10	4
D	9	3	5	10

Who is the best?

What is the problem?

- Given a set $A = \{x, y, z, w, \cdots\}$;
- Given a set of attributes D;
- Given the assessment of A against D;
- Given preference statements of the type $x \succeq_i y$, $x, y \in A$;

Partition the set A in ordered undefined equivalence classes (ranking).

Borda vs. Condorcet

Four candidates and seven examiners with the following preferences.

	a	b	c	d	e	f	g
A	1	2	4	1	2	4	1
B	2	3	1	2	3	1	2
C	3	1	3	3	1	2	3
D	4	4	2	4	4	3	4

Borda vs. Condorcet

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A	1	2	4	1	2	4	1
B	2	3	1	2	3	1	2
C	3	1	3	3	1	2	3
D	4	4	2	4	4	3	4

B(x)
15
14
16
25

The Borda count gives B$>$A$>$C$>$D

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Four candidates and seven examiners with the following preferences.

	a	b	c	d	e	f	g
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C	3	1	2	3	1	2	3

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C	3	1	2	3	1	2	3

B(x)
13
14
15

If D is not there then A>B>C, instead of B>A>C

Borda vs. Condorcet

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The Condorcet principle gives A>B>C>A !!!!

Arrow's Theorem

Given N rational voters over a set of more than 3 candidates can we found a social choice procedure resulting in a social complete order of the candidates such that it respects the following axioms?

- Universality: the method should be able to deal with any configuration of ordered lists;
- Unanimity: the method should respect a unanimous preference of the voters;
- Independence: the comparison of two candidates should be based only on their respective standings in the ordered lists of the voters.

YES!

There is only one solution: the dictator!!

If we add no-dictatorship among the axioms then there is no solution.

Gibbard-Satterthwaite's Theorem

When the number of candidates is larger than two, there exists no aggregation method satisfying simultaneously the properties of universal domain, non-manipulability and non-dictatorship.

Why MCDA is not Social Choice?

Social Choice	MCDA
Total Orders	Any type of order
Equal importance of voters	Variable importance of criteria
As many voters as necessary	Few coherent criteria
No prior information	Existing prior information

Counting values

$$x \succeq y \Leftrightarrow \sum_j r_j(x) \geq \sum_j r_j(y)$$

What do we need to know?

Counting values

$$x \succeq y \Leftrightarrow \sum_j r_j(x) \geq \sum_j r_j(y)$$

What do we need to know?

the primitives: $\succeq_j \subseteq A \times A$
Differences of preferences:
- $(xy)_1 \succcurlyeq (zw)_1$
- $(xy)_1 \succcurlyeq (zw)_2$

How do we learn that?

- Directly through a standard protocol.
- Indirectly:
  - through pairwise comparisons (AHP, MACBETH etc.);
  - through learning from examples (regression, rough sets, decision trees etc.).

Is this sufficient?

NO!

Are preferences independent?

$r \succ w$

$f \succ m$

But *rf* is not better than *wf* ...

Non linear aggregation procedures

What is the output?

- Value functions on each criterion.
- A global value function.
- Rankings, choices, but also ratings if relevant reference points are provided on the value function.

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The Value Functions Hypotheses

Collective Rationality

It makes no sense to try to fix society's preferences (Arrow's theorem). Instead we can look to model a specific decision maker/stakeholder preferences since she could have consistent values.

Subjective Values

If this is true then we can try to "measure" the consequences of any project or policy against such values: this is a subjective value function.

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How do we measure better?

Let's go more formal.

- Let $x, y, z \ldots$ be competing projects within set A;
- Let $d_j(x)$ representing the consequences of project x on dimension d_j;
- Let $d_j(A)$ representing the set of all consequences for all projects in A.

The first step consists in verifying that:

$$\forall j \in D \ \exists \succeq_j \subseteq d_j(A)^2$$

such that $\succeq_j$ is a weak order (consequences should be completely and transitively ordered).

How do we measure better?

If the previous hypothesis is verified then

$$\forall j \in D \;\; \exists h_j : A \mapsto \mathbb{R} : d_j(x) \succeq d_j(y) \Leftrightarrow h_j(x) \geq h_j(y)$$

In other terms for each dimension we can establish a real valued function respecting the decision maker's preferences.

This function is ONLY an ordinal measure of the preferences

Example-1

Suppose you have 4 projects x, y, z, w of urban rehabilitation and an assessment dimension named “esthetics”. You have:
- $d_e(x) =$ statue;
- $d_e(y) =$ fountain;
- $d_e(z) =$ garden;
- $d_e(w) =$ kid’s area;

Preferences expressed could be for instance:
$d_e(x) \succ d_e(y) \succ d_e(z) \sim d_e(w)$
A possible numerical representation could thus be:
$h_e(x) = 3$, $h_e(y) = 2$, $h_e(z) = h_e(w) = 1$

Example-2

Suppose you have 4 projects x, y, z, w of urban rehabilitation and an assessment dimension named “land use”. You have:
- $d_l(x) = 100$sqm;
- $d_l(y) = 50$sqm;
- $d_l(z) = 1000$sqm;
- $d_l(w) = 500$sqm;

Preferences expressed could be for instance (suppose the decision maker dislikes land use:
$d_e(y) \succ d_e(x) \succ d_e(w) \sim d_e(z)$
A possible numerical representation could thus be:
$h_e(y) = 4$, $h_e(x) = 3$, $h_e(w) = 2$ $h_e(z) = 1$, but also:
$h_e(y) = 50$, $h_e(x) = 100$, $h_e(w) = 500$ $h_e(z) = 1000$

Is this sufficient?

For the time being we have the following table:

	d_1-h_1	d_2-h_2	$\dots$	d_n-h_n
x				
y				
z				
w				
$\vdots$				

The consequences of each action and the numerical representation of the decision maker's preferences (ordinal).

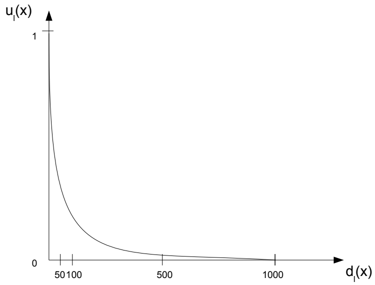
Is this sufficient?

NO!

We need something more rich. We need to know, when we compare x to y (and we prefer x) if this preference is “stronger” to the one expressed when comparing (on the same dimension) z fo w.

We need to compare differences of preferences

An example



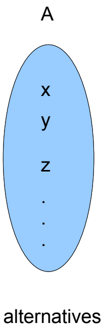
For instance, if the above function represents the value of "land use" it is clear that the difference between 50sqm and 100sqm is far more important from the one between 500sqm and 1000sqm.

First Summary

Let's summarise our process until now.

- We get the alternatives.
- We identify their consequences for all relevant dimensions.
- These consequences are ordered for each dimension using the decision maker's preferences.
- We compute the value function measuring the differences of preferences (for each dimension).

First Summary



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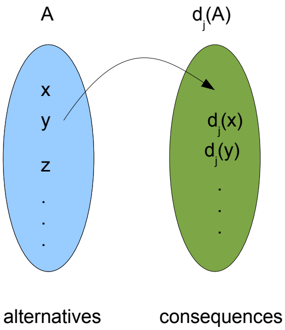

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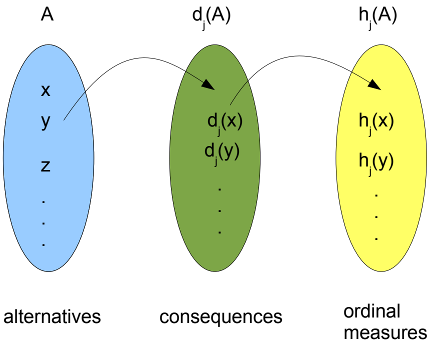

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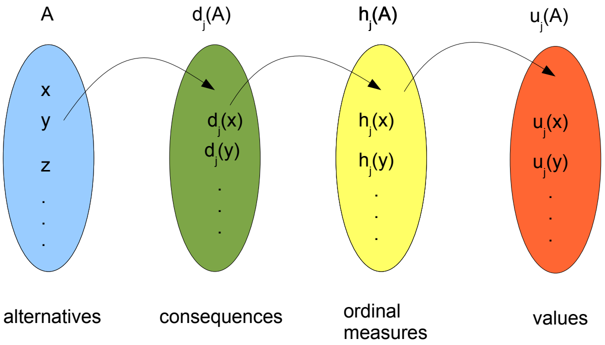

Is all that sufficient?

NO!

1. The problem is that we need to be able to compare the differences of preferences on one dimension to the differences of preferences on another one (let's say differences of preferences on land use with differences of preferences on esthetics.
2. At the same time we need to take into account the intuitive idea that for a given decision maker certain dimensions are more "important" than other ones.

Principal Hypotheses

1. The different dimensions are separable.
2. Preferences on each dimension are independent.
3. Preferences on each dimension are measurable in terms of differences.
4. Good values on one dimension can compensate bad values on another dimension.

Principal Hypotheses

Under the previous hypotheses we can construct a global value function $U(x)$ as follows:

$$U(x) = \sum_j u_j(x)$$

and in case we use normalised (in the interval [0,1]) marginal value functions $\bar{u}_j$ then:

$$U(x) = \sum_j w_j \bar{u}_j(x)$$

Principal Hypotheses

where: w_j should represent the importance of the marginal functions;
If $h_j(x)$ represent the ordinal values of dimension j then $u_j(d_j(\underline{x})) = 0$ where $d_j(\underline{x})$ is the worst value of h_j
and in case we use normalised value functions then $u_j(d_j(\overline{x})) = 1$ where $d_j(\overline{x})$ is the best value of h_j.

Standard Protocol

1. Fix arbitrary one dimension as the reference for which the value function will be linear (there is no loss of generality doing so).
2. Fix a number of units diving entirely the reference value function, thus fixing the unit of value U_1.
3. Une indifference questions (see later) in order to find equivalent values for the other dimensions.
4. The segments between the equivalent values will shape the other value functions.
5. The ratio of units used to describe each value function with respect to the units for the reference one establishes the trade-offs among the dimensions.

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Indifference Questions

Given d_r as the reference dimension, h_r being the ordinal preferences we want to establish a value function for dimension d_k. Consider a fictitious object x for which we have $\langle h_r(x), h_k(x)\rangle$. The key question is:

$$\langle h_r(x), h_k(x)\rangle \sim \langle h_r(\bar{x}), ?\rangle$$

What should be the measure on dimension k of an object $\bar{x}$ whose measure on the reference dimension r is such that the $u_r(\bar{x}) = u_r(x) + U_1$ if x and $\bar{x}$ should be indifferent for the decision maker?

Indifference Questions

Once you get the answer $h_k(\bar{x})$ from the decision maker you go ahead:

$$\langle h_r(x), h_k(\bar{x})\rangle \sim \langle h_r(\bar{x}), ?\rangle \quad \rightarrow h_k(\bar{\bar{x}})$$

$$\langle h_r(x), h_k(\bar{\bar{x}})\rangle \sim \langle h_r(\bar{x}), ?\rangle \quad \rightarrow h_k(\bar{\bar{\bar{x}}})$$

Until the whole set of measures of dimension k has been used.

TIPS

- **TIP1** Start considering a point x at the middle of both scales h_r and h_k.
- **TIP2** Then start deteriorating on the reference dimension by one unit of value at time (thus the dimension under construction has to improve) until the upper scale of h_k is exhausted.
- **TIP2** Then start improving on the reference dimension by one unit of value at time (thus the dimension under construction has to deteriorate) until the lower scale of h_k is exhausted.

What do we get?

We have $U(x) = u_r(x) + u_k(x)$ by definition.
We also have $U(\bar{x}) = u_r(\bar{x}) + u_k(\bar{x})$ after questioning.
And since x and $\bar{x}$ are considered indifferent $U(x) = U(\bar{x})$.
Then we get $u_r(x) + u_k(x) = u_r(x) + U_1 + u_k(\bar{x})$ by construction.
We obtain $u_k(\bar{x}) = u_k(x) - U_1$.

Going ahead recursively we found the point $\underline{x}$ at the bottom of the scale for which by definition $u_k(\underline{x}) = 0$ (by definition). Using linear segments between all the points discovered we shape the value function u_k.

Example

You have to choose among competitive projects assessed against 3 attributes: cost, esthetics and mass. As far as the cost is concerned the scale goes from 5M€ to 10M€. Esthetics are assessed on a subjective scale going from 0 to 8. Mass is measured in kg and the scale goes from 1kg to 5kg. In this precise moment you have under evaluation the following four ones:

project	c	e	m
A	6,5M€	3	3kg
B	7,5M€	4	4,5kg
C	8M€	6	2kg
D	9M€	7	1,5kg

Which is the “best choice”?

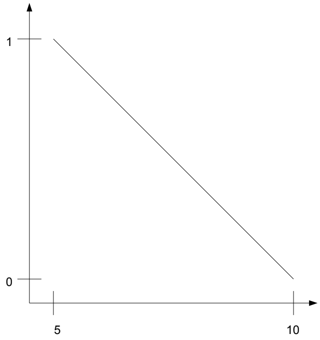
Preferences

First we need to establish appropriate preferences. Suppose in your case the following ones:

- you prefer the less expensive to the more expensive (cost);
- you prefer “pretty” to “less pretty” (esthetics);
- you prefer “heavy” to “less heavy” (mass).

Cost Value Function

Without loss of generality we establish the cost as reference criterion with a linear value function such that $u_c(5M€) = 1$ and $u_c(10M€) = 0$. We fix the value unit $U_1 = 0{,}5M€$.



Cost Value Function

Esthetics Value Function

In order to construct the value function of Esthetics we proceed with the following dialog:

$$\langle 7.5\text{M}€, 4\rangle \sim \langle 8\text{M}€, ?\rangle$$

Consider a project which costs 7.5 € and is assessed on esthetics with 4, and a project which costs 8M € (one unit of value less in this case), how much should the second project be improved in esthetics in order to be indifferent to the first one?
Suppose we get an answer of 5: $\langle 7.5\text{M}€, 4\rangle \sim \langle 8\text{M}€, 5\rangle$
We repeat now the question using the new value:

$$\langle 7.5\text{M}€, 5\rangle \sim \langle 8\text{M}€, ?\rangle$$

We now get an answer of 6.

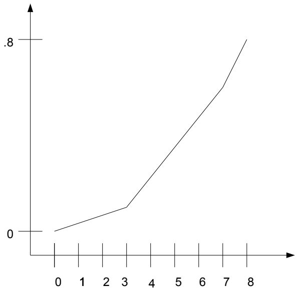
Esthetics Indifferences

We can summarise the dialog as follows:

$$\langle 7.5\text{M}€, 4\rangle \sim \langle 8\text{M}€, 5\rangle$$
$$\langle 7.5\text{M}€, 5\rangle \sim \langle 8\text{M}€, 6\rangle$$
$$\langle 7.5\text{M}€, 6\rangle \sim \langle 8\text{M}€, 7\rangle$$
$$\langle 7.5\text{M}€, 7\rangle \sim \langle 8\text{M}€, 7.5\rangle$$
$$\langle 7.5\text{M}€, 7.5\rangle \sim \langle 8\text{M}€, 8\rangle$$
$$\langle 7.5\text{M}€, 4\rangle \sim \langle 7\text{M}€, 3\rangle$$
$$\langle 7.5\text{M}€, 3\rangle \sim \langle 7\text{M}€, 1.5\rangle$$
$$\langle 7.5\text{M}€, 1.5\rangle \sim \langle 7\text{M}€, 0\rangle$$

Esthetics Value Function

The previous dialog will result in the following value function.



Esthetics Value Function

Mass Value Function

In order to construct the value function of Mass we proceed with the following dialog:

$$\langle 7.5\text{M}€, 3.1\rangle \sim \langle 8\text{M}€, ?\rangle$$

Consider a project which costs 7.5€ and weighs 3.1kg and a project which costs 8M€ (one unit of value less in this case), how much should the second project be improved in mass in order to be indifferent to the first one? Suppose we get an answer of 3.5kg: $\langle 7.5\text{M}€, 3.1\rangle \sim \langle 8\text{M}€, 3.5\rangle$
We repeat now the question using the new value:

$$\langle 7.5\text{M}€, 5\rangle \sim \langle 8\text{M}€, ?\rangle$$

We now get an answer of 3.9.

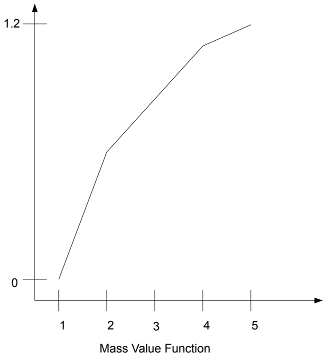
Mass Indifferences

We can summarise the dialog as follows:

$$
\begin{gathered}
\langle 7.5M€, 3.1\rangle \sim \langle 8M€, 3.5\rangle \\
\langle 7.5M€, 3.5\rangle \sim \langle 8M€, 3.9\rangle \\
\langle 7.5M€, 3.9\rangle \sim \langle 8M€, 5\rangle \\
\langle 7.5M€, 3.1\rangle \sim \langle 7M€, 2.7\rangle \\
\langle 7.5M€, 2.7\rangle \sim \langle 7M€, 2.3\rangle \\
\langle 7.5M€, 2.3\rangle \sim \langle 7M€, 1.9\rangle \\
\langle 7.5M€, 1.9\rangle \sim \langle 7M€, 1.75\rangle \\
\langle 7.5M€, 1.75\rangle \sim \langle 7M€, 1.6\rangle \\
\langle 7.5M€, 1.6\rangle \sim \langle 7M€, 1.45\rangle \\
\langle 7.5M€, 1.45\rangle \sim \langle 7M€, 1.3\rangle \\
\langle 7.5M€, 1.3\rangle \sim \langle 7M€, 1.15\rangle \\
\langle 7.5M€, 1.15\rangle \sim \langle 7M€, 1\rangle
\end{gathered}
$$

Mass Value Function

The previous dialog will result in the following value function.



Final calculations

Having obtained the three value functions we can now calculate the values of the four projects for each of them.

$$
\begin{array}{lll}
u_c(A) = 0.7 & u_e(A) = 0.2 & u_m(A) = 0.875 \\
u_c(B) = 0.5 & u_e(B) = 0.3 & u_m(B) = 1.160 \\
u_c(C) = 0.4 & u_e(C) = 0.5 & u_m(C) = 0.625 \\
u_c(D) = 0.2 & u_e(D) = 0.6 & u_m(D) = 0.330
\end{array}
$$

Final Results

Finally we get

$$
\begin{aligned}
U_c(A) &= 0.7 + 0.2 + 0.875 = 1.775\\
U_c(B) &= 0.5 + 0.3 + 1.160 = 1.960\\
U_c(C) &= 0.4 + 0.5 + 0.625 = 1.525\\
U_c(D) &= 0.2 + 0.6 + 0.330 = 1.130
\end{aligned}
$$

The project which maximises the decision maker's value is B.

Where did the weight disappear?

NOWHERE

Suppose we were using normalised value functions which have to be "weighted". We recall that in such a case we have:

$$U(x) = \sum_j w_j \bar{u}_j(x)$$

Consider the first indifference sentence about esthetics. We had: $\langle 7.5\text{M€}, 4\rangle \sim \langle 8\text{M€}, 5\rangle$. We get:
$w_c\bar{u}_c(7.5\text{M€}) + w_e\bar{u}_e(4) = w_c\bar{u}_c(8\text{M€}) + w_e\bar{u}_e(5)$
where:
- w_c and w_e represent the "weights" of cost and esthetics respectively;
- and $\bar{u}_c$ and $\bar{u}_e$ are the normalised value functions.

Here are the weights ...

By construction $u_c(x) = \bar{u}_c(x)$. We get:
$w_c(\bar{u}_c(7.5\text{M€}) - \bar{u}_c(8\text{M€})) = w_e(\bar{u}_e(5) - \bar{u}_e(4))$. Thus:

$$\frac{w_e}{w_c} = \frac{\bar{u}_c(7.5\text{M€}) - \bar{u}_c(8\text{M€})}{\bar{u}_e(5) - \bar{u}_e(4)}$$

However, $\bar{u}_c(7.5\text{M€}) - \bar{u}_c(8\text{M€}) = 1/10$ of the cost value function (by construction) and $\bar{u}_e(5) - \bar{u}_e(4) = 1/8$ of the esthetics value function as it results from the dialog. Using the same procedure for mass we get:

- $w_e/w_c = 0.8$ meaning that esthetics represents 80% of the cost value (this is the esthetics trade-off);
- $w_m/w_c = 1.2$ meaning that mass represents 120% of the cost value (this is the mass trade-off);

Conclusion and tips

Tip1 Not surprisingly the "weight" of each criterion is represented by the maximum value it attains.

Tip2 It is better not to use any "weights" when constructing value functions, since it can generate confusion to the decision maker. We can explain the relative importance of each criterion using the trade-offs.

So called "weights" are the trade-offs among the value functions and as such are established as soon as the value functions are constructed. They do not exist independently and is not correct to ask the decision maker to express them.

Counting preferences

$$x \succeq y \;\Leftrightarrow\; H_{xy} \geq H_{yx}$$

What do we need to know?

Counting preferences

$$x \succeq y \Leftrightarrow H_{xy} \geq H_{yx}$$

What do we need to know?

the primitives: $\succeq_j \subseteq A \times A$
An ordering relation on $2^{\succeq_j}$

How do we learn that?

- Preferences are “given”.
- Preferences on $2^{\succeq_j}$:
  - directly;
  - coalition games;
  - learning from examples.

Is this sufficient?

NO!

- The relation $\succeq$ is not an ordering relation.
- We need to construct an ordering relation $\succcurlyeq$ "as near as possible" to $\succeq$.
- In order to do so we transform the graph induced by $\succeq$.

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General idea: coalitions

Given a set A and a set of $\succeq_i$ binary relations on A (the criteria) we define:

$$x \succeq y \Leftrightarrow C^+(x,y) \trianglerighteq C^+(y,x) \text{ and } C^-(x,y) \trianglelefteq C^-(y,x)$$

where:
- $C^+(x,y)$: "importance" of the coalition of criteria supporting x wrt to y.
- $C^-(x,y)$: "importance" of the coalition of criteria against x wrt to y.

How it works? 1

Additive Positive Importance

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$$C^+(x,y) = \sum_{j \in J^{\pm}} w_j^+$$

where:
w_j^+: “positive importance” of criterion i
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Where "positive importance" comes from?

How it works? 2

Max Negative Importance

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$$C^{-}(x, y) = \max_{j \in J^{-}} w_j^{-}$$

where:

w_j^{-}: "negative importance" of criterion i

$J^{-} = \{h_j : v_j(x, y)\}$

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Max Negative Importance

$$C^-(x,y) = \max_{j \in J^-} w_j^-$$

where:
w_j^-: "negative importance" of criterion i
$J^- = \{h_j : v_j(x,y)\}$

Then we can fix a veto threshold γ and have

$$x \succeq^- y \Leftrightarrow C^-(x,y) \geq \gamma$$

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w_j^{-}: "negative importance" of criterion i
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Then we can fix a veto threshold γ and have

$$x \succeq^{-} y \Leftrightarrow C^{-}(x,y) \geq \gamma$$

Where "negative importance" comes from?

Example

The United Nations Security Council

Positive Importance

15 members each having the same positive importance $w_j^+ = \frac{1}{15}$, $\delta = \frac{9}{15}$.

Negative Importance

10 members with 0 negative importance and 5 (the permanent members) with $w_j^- = 1$, $\gamma = 1$.

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Outranking Principle

$$x \succeq y \Leftrightarrow x \succeq^{+} y \text{ and } \neg(x \succeq^{-} y)$$

Thus:

$$x \succeq y \Leftrightarrow C^{+}(x,y) \geq \delta \wedge C^{-}(x,y) < \gamma$$

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NB

The relation $\succeq$ is not an ordering relation. Specific algorithms are used in order to move from $\succeq$ to an ordering relation $\succcurlyeq$

What is importance?

Where w_j^+, w_j^- and δ come from?

Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.

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Example

Given a set of criteria and a set of decisive coalitions ($J^{\pm}$) we can solve:

$$\max \delta$$
$$\text{subject to}$$
$$\sum_{j \in J^{\pm}} w_j \geq \delta$$
$$\sum_j w_j = 1$$

And the final ranking?

- $x \succcurlyeq y \;\Leftrightarrow\; o(x) - i(x) \geq o(y) - i(y)$
- Recursively constructing $\succcurlyeq$:
  - $[x]_1 = \{x \in A : \; \neg \; \exists y \;\; y \succeq x\}$
    $[x]_i = \{x \in A \setminus \cup_{i-1}[x] : \; \neg \; \exists y \;\; y \succeq x\}$
  - $[x]_n = \{x \in A : \; \neg \; \exists y \;\; x \succeq y\}$
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Rating

What if we have preference relations $\succeq_j \subseteq A \times P \cup P \times A$? The global preference relation remains the same.

- pessimistic rating
  - x is iteratively compared with $p_t \cdots p_1$,
  - as soon as $x \succeq p_h$) is established, assign x to category c_h.
- optimistic rating
  - x is iteratively compared with $p_1 \cdots p_t$,
  - as soon as is established $p_h \succeq x) \wedge \quad \neg \quad x \succeq p_h$) then assign x to category c_{h-1}.

What is the output?

- A global preference relation including incomparabilities.
- An explicit representation of hesitation.
- Robust Rankings, Choices and Ratings.

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Lessons Learned

- We can use social choice inspired procedures for more general decision making processes.
- Care should be taken to model the majority (possibly the minority) principle to be used. The key issue here is the concept of "decisive coalition".
- We need to "learn" about decisive coalitions, since it is unlike that this information is available. Problem of learning procedures.
- The above information is not always intuitive. However, the intuitive idea of importance contains several cognitive biases.
- A social choice inspired procedure will not deliver automatically an ordering. We need further algorithms (graph theory).

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Resources

- http://www.algodec.org
- http://www.cs.put.poznan.pl/ewgmcda/
- http://www.decision-deck.org
- http://decision-analysis.society.informs.org/
- http://www.mcdmsociety.org/
- http://www.euro-online.org
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Books

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