Preferences extension rules and coalitional games

communique par Stefano Moretti (stefano.moretti@dauphine.fr)
et Alexis Tsoukis (Alexis.Tsoukias@lamsade.dauphine.fr)

A lot of problems in individual and collective decision making involve the comparison of *sets of alternatives* (e.g., comparing the outcomes of social choice correspondences; non probabilistic models of choice under uncertainty; coalition formations, etc.). However, a ranking of the single alternatives in a finite set N is not sufficient to compare the subsets of N. How to 'properly' infer preferences that are capable of ranking subsets of N?

Many papers on this problem have mostly focused on the question of how one can construct an ordering over subsets of N given an ordering over the elements of N (Fishburn (1992), Barberà et al. (2004), Brewka et al. (2010)). More precisely, they have studied the problem of *extending* a (complete) pre-order over a set to its powerset. For instance, some authors have analyzed the axiomatic structure of families of rankings over subsets (Kannai and Peleg (1984), Barbera, Barret and Pattanaik (2004), Fishburn (1992), Bossert (1995), Alcantud and Arlegi (2008), etc.).

To better illustrate the problem, consider three objects a, b and c, representing, respectively, an '$a$pple', a '$b$anana' and a piece of '$c$hocolate'. To be concrete, suppose that a is preferred to b, and b is preferred to c. What should the relative ranking of the subsets of $\{a, b, c\}$ be? There is, of course, no single 'right' answer to this question because the answer will depend on the context of the situation, on the interpretation of the ranking of sets in that context, and on the criteria used to establish this ranking.

For example, it could seem quite natural to assume that $\{a, b\}$ is preferred to $\{a, c\}$. *Why*? It is sufficient to assume that we are using an extension procedure which satisfies the *Responsiveness* property (Bossert (1995)): take S subset of N and consider an element x in S and another one y which is not in S, and assume that x is preferred to y. Now, replace x with y in S: the new set we have is less 'appealing' then S, since x was preferred to y (in the example, take $\{a, b\}$ in the role of S, b and c in the role of x and y, respectively).

But it could be the case that, due to the combination of tastes, 'apple and chocolate' ($\{a, c\}$) is my favorite dessert... maybe my favorite food!!

Differently stated, the responsiveness property suggests that *interactions* among elements of N in the sets to be compared are not very important in establishing the ranking among sets. Then, is it so natural that an extension method satisfies such a property? Alternatively, we could explore the possibility to consider properties for extension methods that are oriented to capture aspects related to elements' interaction. Following this approach, that is the *property-driven* approach (also called, the *axiomatic* approach), the interpretation of properties plays a central role.

The axiomatic approach has been (even more) important for another (apparently) unrelated research field: *game theory* (von Neumann and Morgenstern (1944)). Over the last sixty years, game theory has been widely used to analyze the interactions between several agents (which are called *players*). In particular, *cooperative* games have been widely applied to model situations where agents may form *coalitions* (Owen (1995)). Among other solutions concepts for cooperative games, the *Shapley value* for *coalitional games* (Shapley(1953)) has been successfully applied in many different contexts to convert information about the worth that subsets of the player set achieve, into a personal attribution (of payoff) to each of the players (Moretti and Patrone (2008)). Our main objective is to look at these models to understand whether they can be useful to attack the problem of defining methods for preference extension that keep into account the interaction among elements.

We aim at exploring this issue focusing on the following tasks.

Minor tasks:

1) to look at the literature of preference extension models using the axiomatic approach, with the objective to classify the properties used on the basis of their interpretation. In particular, the main goal will be to find properties whose interpretation is oriented to model the interaction among alternatives.

2) to survey extension methods introduced in literature, focusing on those methods which deal with situations where some subsets of N are formed by elements that are incompatible.

Major tasks:

1) beside the possibility to compare the axioms which characterize alternative approaches, it seems not completely clear how to operatively compare methods aimed to construct an ordering over subsets on particular instances. Which is the role of elements' interaction in extending a ranking over all possible subsets? Is it possible to provide an overall measure of the level of interaction among the elements of N, when a certain procedure which extends a numerical representation of preferences on N to its powerset is adopted?

2) can the Shapley value (of a coalitional game representing a ranking over all possible subsets of N) be applied as an index to measure the level of interaction among elements when a method extending a ranking over N to its powerset is used?

3)is it possible (and meaningful) to characterize those methods for preference extension which determine coalitional games whose Shapley value preserves the original ranking over the elements of N?

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