Methods and Tools for Public Policy Evaluation

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Outline

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1. Introduction
   - Public Decision Processes
   - What is Evaluation?
   - Decision Aiding
2. Basics
3. Cost-Benefit Analysis
4. Multi-attribute Value Functions
5. Further Reading

A legitimation issue

Policy makers feel lacking legitimation in their policy making process.

- Mistrust between public opinion, experts and policy makers.
- Information society and information circulation and availability.
- Social fragmentation.
- Short agendas vs. long term concerns.

What is a public decision process?

- Distributed Decision Power (several stakeholders).
- Different Rationalities.
- Participation “de facto”.
- Public Deliberation.
- Social Outcomes.
- Long Time Horizon

Specificities

What is specific in Public Policies?

- Different types of Actors:
  - Political actors (short term political agendas).
  - Officials and Experts (medium term knowledge based agendas).
  - Social groups more or less fragmented.
- Different types of stakes.
  - From long term and/or affecting large parts of territory and population, to
  - short term individual “opportunistic” stakes.
- Heterogeneous resources such as: knowledge, trust, money, land, authority, power etc. are committed in the process.

Consequences

What are the consequences?

- Conflicting opinions, priorities, actions.
- Conflicting information and interpretations.
- Different languages and communication patterns.
- Mutually adaptive behaviour along time.

What does it mean?

Accountability, Legitimation, Consensus, Evidence

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Formal models in Public Policy Assessment

Advantages

- Common language.
- Improved accountability.
- Basis for participative decision making.
- Exploring less "obvious aspects" (better insight).
- Avoiding intuitive errors.

Formal models in Public Policy Assessment

Drawbacks

- Possible loss of a global insight.
- Possible loss of creative thinking.
- Too much structuring of the decision process.
- Does everybody understands formal models?
- Cost of using formal models.

Outline

What is Evaluation?

- What does it mean evaluation?
- *Measuring value*
- What does it mean measuring?
- What is value?

Values?

- Value of what?
- Value for whom?
- Value for doing what?
- Is there an objective value?

Values?

- Value as a social agreement.
- Economic value and money.
- Value of use and marginal value.
- Personal values.
- Values as ethics and norms.

Did the air quality improved?

pollutant	CO_2	SO_2	O_3	dust
t_1	3	3	8	8
t_2	3	3	8	2

Did the air quality improved?

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t_1	3	3	8	8
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The air quality improved, but for the ATMO index did not. Who tells the truth?

Meaningfulness

Theoretical Soundness

Information needs to be manipulated in a coherent and consistent way (measurement theory).

Operational Completeness

Information needs to be manipulated in order to be useful for who is using it and for those purposes for which has been designed. It should allow to reach a conclusion.

What does it mean?

95% of rural households in Burkina Faso do not have tap water available

- For us this is a serious problem and evidence of poverty, but for the locals is not.
- For the local men this is not a problem, while it is for the local women.

Differences of perspective

- Different standards and thresholds.
- Different cultures.
- Different stakeholders.
- Different concerns.
- Different resources.

Is it good or bad?

The h-index of X is 19. Is (s)he a good researcher?

- Who is a good researcher?
- What good research means?
- Who decides and for what purpose about research quality?

Values

What do we take into account?

- Values and preferences of relevant stakeholders.
- Individual values and social values.
- Judgements (experts, politicians, opinions).

Who is the winner?

10 voters have preferences $aPbPc$,
6 voters have preferences $bPcPa$
and 5 voters have preferences $cPbPa$.

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Most electoral systems will choose *a*, which is the one the majority does not want. Actually the Condorcet winner is *b*.

Different ways to construct evidence

- Different ways to establish a majority.
- Different ways to compute an average.
- Different ways to take into account the importance of ...
- Positive and Negative reasons/arguments.

Evaluation and Decision Aiding

Not easy ...

Evaluating is less easy intuitive from what it appears to be

... to aid to decide

Evaluating is a Decision Aiding activity

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Evaluating is a Decision Aiding activity

Outline

Deciding ...

- Decision Maker
- Decision Process
- Cognitive Effort
- Responsibility
- Decision Theory

... and Aiding to Decide

- A client and an analyst
- Decision Aiding Process
- Cognitive Artifacts
- Consensus
- Decision Aiding Methodology

What is a Decision Aiding Process?

The interactions between somebody involved in a decision process (the client) and somebody able to support him/her within the decision process.

Consensual construction of shared cognitive artifacts

A Decision Aiding Process makes sense only with respect to a Decision Process in which the client is involved and with respect to which demands advice.

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What is a Decision Aiding Process?

A Decision Aiding Process is a Decision Process where at least two actors are involved: the client and the analyst, with at least two concerns: the client's "problem" and the analyst's job, mobilising at least the following resources: the client's domain knowledge and the analyst's methodological knowledge.

A Decision Aiding Process becomes part of the Decision Process for which it has been established. The analyst enters as an actor such a Decision Process.

Its Cognitive Artifacts

- Representation of the problem situation
- Problem Formulation
- Evaluation Model
- Final Recommendation

Representing a Problem Situation

- Who has a problem?
- Why this is a problem?
- Who other is affected by the decision process?
- Who decides?
- Who pays for the consequences and the bill?
- What I am doing here?

A Problem Situation

$$PS = \langle \mathcal{A}, \mathcal{O}, RS \rangle$$

- $\mathcal{A}$ actors, participants, stakeholders
- $\mathcal{O}$ objects, concerns, stakes
- RS resources, commitment

Formulating a Problem

Constructing a first formal representation of the client's concerns, applying an abstract and formal language, using a model of rationality.

- What objects do we consider in formulating "the problem"?
- What do we know or are we looking for such objects?
- What do we want to do with such objects?

A Problem Formulation

$$\Gamma = \langle \mathbb{A}, \mathbb{V}, \Pi \rangle$$

- $\mathbb{A}$ Actions
- $\mathbb{V}$ Points of view
- Π Problem statement

Constructing an Evaluation Model

- Fixing alternatives.
- How to describe them?
- Are there any preferences?
- Are we sure about the information?
- How to put all this information together?

Evaluation Model

$$\mathcal{M} = \langle A, D, E, H, \mathcal{U}, \mathcal{R} \rangle$$

- A alternatives, decision variables, ...
- D dimensions, attributes, ...
- E scales associated to attributes,
- H criteria, preference models, ...
- $\mathcal{U}$ uncertainty, epistemic states, ...
- $\mathcal{R}$ procedures, algorithms, protocols ...

Establishing a final Recommendation

- Going back to reality.
- What do we put in the final report?
- Is it valid?
- Is it legitimated?
- It works?
- Are we satisfied?

Meaningfulness ...

- Do we use the information correctly?
- Is it meaningful for the analyst?
  (Measurement Theory)
- Does it make sense for the decision process?
- Is it meaningful for the client?
  (Client Satisfaction)

... and Legitimation

- Ownership
- Organisational Dimension
- Culture
- Decision Process

Notation

- Sets: $A, B \ldots$ of cardinality $n, m, k \ldots$
- Variables $x, y, z \ldots$
- Numbers $\mathbb{N}, \mathbb{Z}, \mathbb{R}$
- Vector Spaces $\mathbb{N}^n, \mathbb{R}^n$
- Binary Relations $\succeq, \succ, \sim$ possibly subscribed
- The usual logical notation $\wedge$, $\vee$, $\rightarrow$, $\neg$, $\forall, \exists$

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What are the problems?

- How to learn preferences?
- How to model preferences?
- How to aggregate preferences?
- How to use preferences for recommending?

Binary relations

- $\succeq$: binary relation on a set (A).
- $\succeq \subseteq A \times A$ or $A \times P \cup P \times A$.
- $\succeq$ is reflexive.

What is that?

If $x \succeq y$ stands for x is at least as good as y, then the asymmetric part of $\succeq$ ($\succ$: $x \succeq y \ \wedge \ \neg(y \succeq x)$) stands for strict preference. The symmetric part stands for indifference ($\sim_1$: $x \succeq y \ \wedge \ y \succeq x$) or incomparability ($\sim_2$: $\neg(x \succeq y) \ \wedge \ \neg(y \succeq x)$).

More on binary relations

- We can further separate the asymmetric (symmetric) part in more relations representing hesitation or intensity of preference.

$$\succ = \succ_1 \cup \succ_2 \cdots \succ_n$$

- We can get rid of the symmetric part since any symmetric relation can be viewed as the union of two asymmetric relations and the identity.
- We can also have valued relations such that: $v(x \succ y) \in [0, 1]$ or other logical valuations ...

Binary relations properties

Binary relations have specific properties such as:

- Irreflexive: $\forall x \;\; \neg(x \succ x)$;
- Asymmetric: $\forall x, y \;\; x \succ y \;\rightarrow\; \neg(y \succ x)$;
- Transitive: $\forall x, y, z \;\; x \succ y \;\wedge\; y \succ z \;\rightarrow\; x \succ z$;
- Ferrers; $\forall x, y, z, w \;\; x \succ y \;\wedge\; z \succ w \;\rightarrow\; x \succ w \;\vee\; z \succ y$;

Numbers

One dimension

$$x \succeq y \quad \Leftrightarrow \quad \Phi(u(x), u(y)) \geq 0$$

where:

$\Phi : A \times A \mapsto \mathbb{R}$. Simple case $\Phi(x, y) = f(x) - f(y)$; $\ f : A \mapsto \mathbb{R}$

Many dimensions

$$x = \langle x_1 \cdots x_n \rangle \quad y = \langle y_1 \cdots y_n \rangle$$

$$x \succeq y \quad \Leftrightarrow \quad \Phi([u_1(x_1) \cdots u_n(_n)], [u_1(y_1) \cdots u_n(y_n)] \geq 0$$

More about Φ in Measurement Theory

Numbers

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Preference Structures

A preference structure

is a collection of binary relations $\sim_1, \cdots \sim_m, \succ_1, \cdots \succ_n$ such that:

- they are pair-disjoint;
- $\sim_1 \cup \cdots \sim_m \cup \succ_1 \cup \cdots \succ_n = A \times A$;
- $\sim_i$ are symmetric and $\succ_j$ are asymmetric;
- possibly they are identified by their properties.

$\sim_1, \sim_2, \succ$ Preference Structures

Independently from the nature of the set A (enumerated, combinatorial etc.), consider $x, y \in A$ as whole elements. Then:

If $\succeq$ is a weak order then:

$\succ$ is a strict partial order, $\sim_1$ is an equivalence relation and $\sim_2$ is empty.

If $\succeq$ is an interval order then:

$\succ$ is a partial order of dimension two, $\sim_1$ is not transitive and $\sim_2$ is empty.

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$\sim_1, \sim_2, \succ_1 \succ_2$ Preference Structures

If $\succeq$ is a *PQI* interval order then:

$\succ_1$ is transitive, $\succ_2$ is quasi transitive, $\sim_1$ is asymmetrically transitive and $\sim_2$ is empty.

If $\succeq$ is a pseudo order then:

$\succ_1$ is transitive, $\succ_2$ is quasi transitive, $\sim_1$ is non transitive and $\sim_2$ is empty.

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What characterises such structures?

Characteristic Properties

Weak Orders are complete and transitive relations.
Interval Orders are complete and Ferrers relations.

Numerical Representations

w.o. $\Leftrightarrow \exists f : A \mapsto \mathbb{R} : x \succeq y \leftrightarrow f(x) \geq f(y)$
i.o. $\Leftrightarrow \exists f, g : A \mapsto \mathbb{R} : f(x) > g(x); x \succeq y \leftrightarrow f(x) \geq g(y)$

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More about structures

Characteristic Properties

PQI Interval Orders are complete and generalised Ferrers relations.
Pseudo Orders are coherent bi-orders.

Numerical Representations

PQI i.o. $\Leftrightarrow$ $\exists f, g : A \mapsto \mathbb{R} :$ $f(x) > g(x); x \succ_1 y \leftrightarrow g(x) > f(y); x \succ_2 y \leftrightarrow f(x) > f(y) > g(x)$
p.o. $\Leftrightarrow$ $\exists f, t, g : A \mapsto \mathbb{R} :$ $f(x) > t(x) > g(x); x \succ_1 y \leftrightarrow g(x) > f(y); x \succ_2 y \leftrightarrow g(x) > t(y)$

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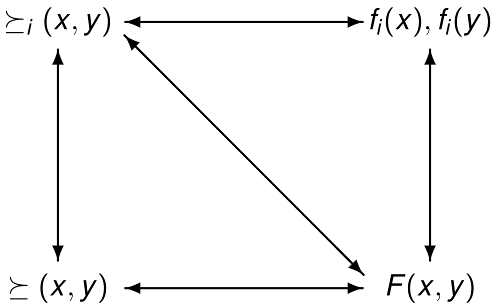
The Problem

- Meaningful numerical representations.
- Putting together numbers (measures).
- Putting together binary relations.
- Overall coherence ...
- Relevance for likelihoods ...

The Problem

Suppose we have n preference relations $\succeq_1 \cdots \succeq_n$ on the set A. We are looking for an overall preference relation $\succeq$ on A "representing" the different preferences.



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What is measuring?

Constructing a function from a set of "objects" to a set of "measures".

Objects come from the real world.

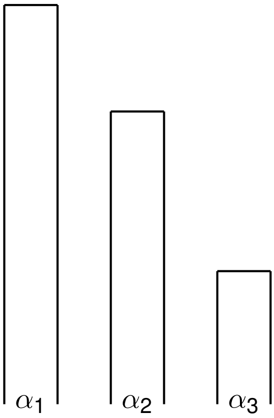
Measures come from empirical observations on some attributes of the objects.

The problem is: how to construct the function out from such observations?

Measurement

1. Real objects $(x, y, \cdots)$.
2. Empirical evidence comparing objects $(x \succeq y, \cdots)$.
3. First numerical representation $(\Phi(x, y) \geq 0)$.
4. Repeat observations in a standard sequence $(x \circ y \succeq z \circ w)$.
5. Enhanced numerical representation $(\Phi(x, y) = \Phi(x) - \Phi(y))$.

Example



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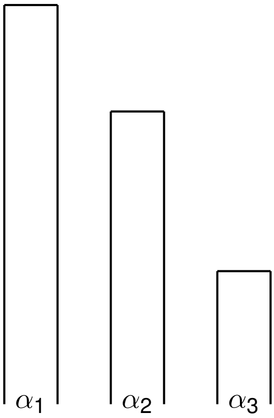

$$\alpha_1 \succ \alpha_2 \succ \alpha_3$$

α_1	α_2	α_3
10	8	6
97	32	12
3	2	1

Any of the above could be a numerical representation of this empirical evidence.

Ordinal Scale: any increasing transformation of the numerical representation is compatible with the EE.

Further Example

Consider putting together objects and observing:

$$\alpha_1 \circ \alpha_5 > \alpha_3 \circ \alpha_4 > \alpha_1 \circ \alpha_2 > \alpha_5 > \alpha_4 > \alpha_3 > \alpha_2 > \alpha_1$$

Consider now the following numerical representations:

	L_1	L_2	L_3
α_1	14	10	14
α_2	15	91	16
α_3	20	92	17
α_4	21	93	18
α_5	28	99	29

L_1, L_2 and L_3 capture the simple order among α_{1-5}, but L_2 fails to capture the order among the combinations of objects.

Further Example

NB

For L_1 we get that $\alpha_2 \circ \alpha_3 \sim \alpha_1 \circ \alpha_4$
while for L_3 we get that $\alpha_2 \circ \alpha_3 > \alpha_1 \circ \alpha_4$.
We need to fix a “standard sequence”.

Length

If we fix a “standard” length, a unit of measure, then all objects will be expressed as multiples of that unit.

Further Example

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Length

If we fix a "standard" length, a unit of measure, then all objects will be expressed as multiples of that unit.

Scales

Ratio Scales

All proportional transformations (of the type αx) will deliver the same information. We only fix the unit of measure.

Interval Scales

All affine transformations (of the type $\alpha x + \beta$) will deliver the same information. Besides the unit of measure we fix an origin.

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More complicated

Consider a Multi-attribute space:

$$X = X_1 \times \cdot X_n$$

to each attribute we associate an ordered set of values:

$$X_j = \langle x_j^1 \cdots x_j^m \rangle$$

An object x will thus be a vector:

$$x = \langle x_1^l \cdots x_n^k \rangle$$

Generally speaking ...

$$x \succeq y$$

$$\Longleftrightarrow$$

$$\langle x_1^l \cdots x_n^k \rangle \succeq \langle y_1^i \cdots y_n^j \rangle$$

$$\Longleftrightarrow$$

$$\Phi(f(x_1^l \cdots x_n^k), f(y_1^i \cdots y_n^j)) \geq 0$$

What that means?

	Commuting Time	Clients Exposure	Services	Size	Costs
a	20	70	C	500	1500

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For what value of δ_1 a and a_1 are indifferent?

What that means?

	Commuting Time	Clients Exposure	Services	Size	Costs
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What that means?

	Commuting Time	Clients Exposure	Services	Size	Costs
a	20	70	C	500	1500
a_1	25	80	C	500	1500
a_2	25	80	C	700	$1500+\delta_2$

What that means?

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a	20	70	C	500	1500
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For what value of δ_2 a_1 and a_2 are indifferent?

What that means?

	Commuting Time	Clients Exposure	Services	Size	Costs
a	20	70	C	500	1500
a_1	25	80	C	500	1500
a_2	25	80	C	700	1700

What that means?

	Commuting Time	Clients Exposure	Services	Size	Costs
a	20	70	C	500	1500
a_1	25	80	C	500	1500
a_2	25	80	C	700	1700

The trade-offs introduced with δ_1 and δ_2 allow to get $a \sim a_1 \sim a_2$

What do we get?

Standard Sequences

Length: objects having the same length allow to define a unit of length;

Value: objects being indifferent can be considered as having the same value and thus allow to define a "unit of value".

Remark 1: indifference is obtained through trade-offs.
Remark 2: separability among attributes is the minimum requirement.

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The easy case

IF

1. restricted solvability holds;
2. at least three attributes are essential;
3. $\succeq$ is a weak order satisfying the Archimedean condition $\forall x, y \in \mathbb{R}, \exists n \in \mathbb{N} : ny > x$.

THEN

$$x \succeq y \Leftrightarrow \sum_j u_j(x) \geq \sum_j u_j(y)$$

General Usage

The above ideas apply also in

- Economics (comparison of bundle of goods);
- Decision under uncertainty (comparing consequences under multiple states of the nature);
- Inter-temporal decision (comparing consequences on several time instances);
- Social Fairness (comparing welfare distributions among individuals).

Outline

Borda vs. Condorcet

Four candidates and seven examiners with the following preferences.

	a	b	c	d	e	f	g
A	1	2	4	1	2	4	1
B	2	3	1	2	3	1	2
C	3	1	3	3	1	2	3
D	4	4	2	4	4	3	4

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C	3	1	3	3	1	2	3
D	4	4	2	4	4	3	4

B(x)
15
14
16
25

The Borda count gives B>A>C>D

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B(x)
13
14
15

If D is not there then A>B>C, instead of B>A>C

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Four candidates and seven examiners with the following preferences.

	a	b	c	d	e	f	g
A	1	2	3	1	2	3	1
B	2	3	1	2	3	1	2
C	3	1	2	3	1	2	3

The Condorcet principle gives A>B>C>A !!!!

Arrow's Theorem

Given N rational voters over a set of more than 3 candidates can we found a social choice procedure resulting in a social complete order of the candidates such that it respects the following axioms?

- Universality: the method should be able to deal with any configuration of ordered lists;
- Unanimity: the method should respect a unanimous preference of the voters;
- Independence: the comparison of two candidates should be based only on their respective standings in the ordered lists of the voters.

YES!

There is only one solution: the dictator!!

If we add no-dictatorship among the axioms then there is no solution.

Gibbard-Satterthwaite's Theorem

When the number of candidates is larger than two, there exists no aggregation method satisfying simultaneously the properties of universal domain, non-manipulability and non-dictatorship.

Why MCDA is not Social Choice?

Social Choice	MCDA
Total Orders	Any type of order
Equal importance of voters	Variable importance of criteria
As many voters as necessary	Few coherent criteria
No prior information	Existing prior information

General idea: coalitions

Given a set A and a set of $\succeq_i$ binary relations on A (the criteria) we define:

$$x \succeq y \Leftrightarrow C^+(x,y) \trianglerighteq C^+(y,x) \text{ and } C^-(x,y) \trianglelefteq C^-(y,x)$$

where:
- $C^+(x,y)$: "importance" of the coalition of criteria supporting x wrt to y.
- $C^-(x,y)$: "importance" of the coalition of criteria against x wrt to y.

Specific case 1

Additive Positive Importance

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$$C^{+}(x,y) = \sum_{j \in J^{\pm}} w_j^{+}$$

where:

w_j^{+}: “positive importance” of criterion i

$J^{\pm} = \{h_j : x \succeq_j y\}$

Specific case 1

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Where "positive importance" comes from?

Specific case 2

Max Negative Importance

Specific case 2

Max Negative Importance

$$C^{-}(x,y) = \max_{j \in J^{-}} w_j^{-}$$

where:
w_j^{-}: "negative importance" of criterion i
$J^{-} = \{h_j : v_j(x,y)\}$

Specific case 2

Max Negative Importance

$$C^{-}(x, y) = \max_{j \in J^{-}} w_j^{-}$$

where:
w_j^{-}: "negative importance" of criterion i
$J^{-} = \{h_j : v_j(x, y)\}$

Then we can fix a veto threshold γ and have

$$x \succeq^{-} y \Leftrightarrow C^{-}(x, y) \geq \gamma$$

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Then we can fix a veto threshold γ and have

$$x \succeq^{-} y \Leftrightarrow C^{-}(x,y) \geq \gamma$$

Where "negative importance" comes from?

Example

The United Nations Security Council

Positive Importance

15 members each having the same positive importance $w_j^+ = \frac{1}{15}$, $\delta = \frac{9}{15}$.

Negative Importance

10 members with 0 negative importance and 5 (the permanent members) with $w_j^- = 1, \gamma = 1$.

Example

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Outranking Principle

$$x \succeq y \Leftrightarrow x \succeq^{+} y \text{ and } \neg(x \succeq^{-} y)$$

Thus:

$$x \succeq y \Leftrightarrow C^{+}(x,y) \geq \delta \wedge C^{-}(x,y) < \gamma$$

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NB

The relation $\succeq$ is not an ordering relation. Specific algorithms are used in order to move from $\succeq$ to an ordering relation $\succcurlyeq$

What is importance?

Where w_j^+, w_j^- and δ come from?

Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.

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Further preferential information is necessary, usually under form of multi-attribute comparisons. That will provide information about the decisive coalitions.

Example

Given a set of criteria and a set of decisive coalitions ($J^{\pm}$) we can solve:

$$\max \delta$$

subject to

$$\sum_{j \in J^{\pm}} w_j \geq \delta$$

$$\sum_j w_j = 1$$

Lessons Learned

- We can use social choice inspired procedures for more general decision making processes.
- Care should be taken to model the majority (possibly the minority) principle to be used. The key issue here is the concept of “decisive coalition”.
- We need to “learn” about decisive coalitions, since it is unlike that this information is available. Problem of learning procedures.
- The above information is not always intuitive. However, the intuitive idea of importance contains several cognitive biases.
- A social choice inspired procedure will not deliver automatically an ordering. We need further algorithms (graph theory).

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Outline

What is Probability?

A measure of uncertainty, of likelihood ...
of subjective belief ...

Consider a set N and a function $p : 2^N \mapsto [0, 1]$ such that:
- $p(\emptyset) = 0$;
- $A \subseteq A \subseteq N$, then $p(A) \leq p(B)$;
- $A \subseteq A \subseteq N, A \cap B = \emptyset$, then $p(A \cup b) = p(A) + p(B)$;

Then the function p is a “probability”.

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A probability is an additive measure of capacity

Decision under risk

	θ_1	θ_2	states of the nature	θ_n
a_1	x_{11}	x_{12}	$\cdots$	x_{1n}
a_2	x_{21}	x_{22}	$\cdots$	x_{2n}
actions	$\cdots$	$\cdots$	outcomes	$\cdots$
a_m	x_{m1}	x_{m2}	$\cdots$	x_{mn}
	p_1	p_2	probabilities	p_n

$$\langle p_1, x_{i1}; p_2, x_{i2}; \cdots p_n, x_{in} \rangle$$

is a lottery associated to action a_i.

Expected Utility

Von Neuman and Morgenstern Axioms

- A1 There is a weak order $\succeq$ on the set of outcomes X.
- A2 If $x \succ y$ implies that $\langle x, P; y, 1-P\rangle \succ \langle x, Q; y, 1-Q\rangle$, then $P > Q$.
- A3 $\langle x, P; \langle y, Q; z, 1-Q\rangle, 1-P\rangle \sim \langle x, P; y, Q(1-P); z, (1-Q)(1-P)\rangle$
- A4 If $x \succ y \succ z$ then $\exists P$ such that $\langle y, 1\rangle \sim \langle x, P; z, 1-P\rangle$

If the above axioms are true then

$$\exists v : X \mapsto \mathbb{R} : \; a_l \succeq a_k \; \Leftrightarrow \; \sum_{j=1}^{n} p_j x_{lj} \geq \sum_{j=1}^{n} p_j x_{kj}$$

Problems

Expected Utility Theory is falsifiable under several points of view

- Gains and losses induce a different behaviour of the decision maker when facing a decision under risk.
- Independence is easily falsifiable.
- Rank depending utilities.
- What happens if probabilities are “unknown”?
- Where probabilities come from?
- What is subjective probability?

Probability does not exist!!!

Ramsey and De Finetti

If the option of α for certain is indifferent with that of β if p is true and γ if p is false, we can define the subject's degree of belief in p as the ratio of the difference between α and γ to that between β and γ (Ramsey, 1930, see also De Finetti, 1936).

Probability does not exist!!!

Ramsey and De Finetti

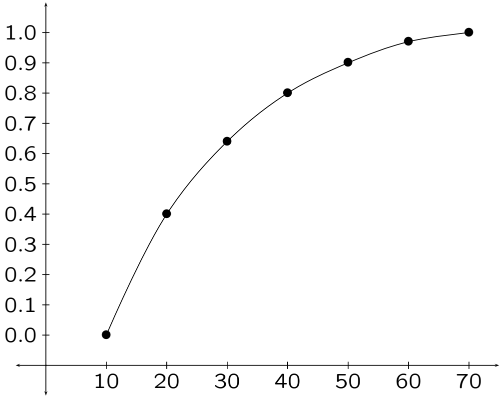
If the option of α for certain is indifferent with that of β if p is true and γ if p is false, we can define the subject's degree of belief in p as the ratio of the difference between α and γ to that between β and γ (Ramsey, 1930, see also De Finetti, 1936).

Savage will give a normative characterisation of von Neuman's expected utility, but the axioms remain empirically falsifiable

How do we build a value function?

Consider the following possible outcomes:
10,20,30,40,50,60,70
Without loss of generality we can consider that:
$v(10) = 0$ and $v(70) = 1$.
Then if the decision maker validates that he is indifferent between a sure outcome of 40 and a lottery $\langle 70, 0.8; 10, 0.2 \rangle$ we get $v(40) = v(70) \times 0.8 + v(10) \times 0.2$, that is $v(40) = 0.8$. With the same protocol we can obtain the following value function.

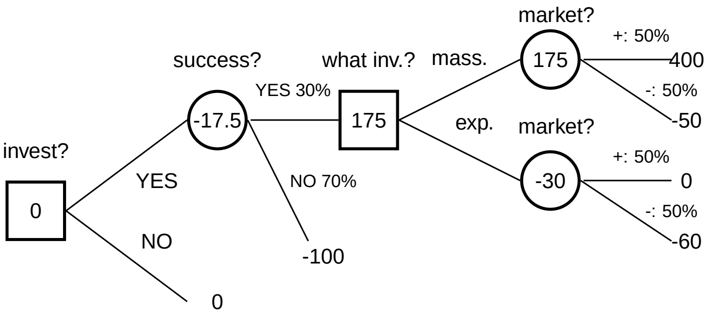
A risk adverse value function



An Example: New Product Development

The R/D department of your company applies for a grant aiming to develop a new "100% fat free" chocolate. They ask for 100K€. There is a 30% probability that they will succeed. If it is the case you face the problem of what type of production you should undertake. If you opt for a mass production and there is a positive reply from the market you can expect 500K€ profit, otherwise the profit will be 50K€. If you make just an experimental production the figures will be respectively 100K€ and 40K€. There is 50% probability that such a product will meet a positive reply from the market. What are your decisions? Consider the monetary outcomes as a value function.

Decision Trees



Decision Trees

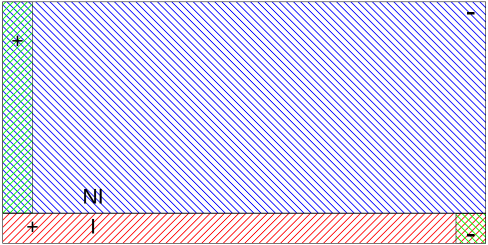
Boxes are decisions. Circles are lotteries.

The value of the lottery “market reaction to mass production” is 175. The value of the lottery “market reaction to exp. production” is -30. The decision therefore is “mass production”. The value of the lottery “success of the R/D” is -17.5, while the no investment has a value of 0. Therefore the decision is not to give the grant to the R/D dept.

Conditional Probabilities

Suppose a serious invalidating illness affecting 1/10000 of the population. There is an examination with 1/100 possibility of error. You undergo such an examination and the result is positive!! What are your chances to be really ill?

Distribution of probabilities



$P(I)$: probability of being ill
$P(NI)$: probability of not being ill
$P(+|I)$: probability of having a positive result if you are ill;
$P(I|+)$: probability of being ill if the result is positive.

$$P(I|+) = \frac{P(I)P(+|I)}{P(I)P(+|I) + P(NI)P(+|NI)} = 0.01$$

Bayes's theorem

Given a set of events $X = \{x_1, \cdots x_n\}$ and the knowledge A then:

$$P(x_k|A) = \frac{P(x_k)P(A|x_k)}{\sum_{i=1\cdots n} P(x_i)P(A|x_i)}$$

Outline

Intuition

Let's start with some intuitive hypotheses.

- An investment should take place only if the expected benefits outperform the expected costs.
- The money used for the investment is either borrowed (from the money market) or if it is used from the investor's treasure it should be at least as profitable as if it was borrowed.

Time horizon

Thus, if K is an investment, then:

$$B(K) - C(K) \geq 0$$

where $B(K)$ ($C(K)$) represent the overall benefits (costs) of the investment (of course we may usual expect to have some profit which implies having a difference more than simply non negative, but for our presentation this is irrelevant).

Fixing a time horizon T (divided in i time periods) within which we may verify the profitability of the investment we get:

$$\sum_{i=1}^{T} B_i(K) - C_i(K) \geq 0$$

Discount Rate

If you borrow 1 € today under an interest rate of r for a period i then at the end of that period you have to return $1 + r$ €.

If you know that at the end of the period you can return X € then at the beginning of that period you can borrow not more than $\frac{X}{1+r}$ €.

If the periods are n then you can borrow at most $\frac{X}{(1+r)^n}$ €.

NPV

On this basis the real value of the investment has to be discounted to the interest rate to which the money is borrowed (at net of the inflation rate if any). If such a discount rate is named r we get:

$$\sum_{i=1}^{T} \frac{B_i(K) - C_i(K)}{(1+r)^n} \geq 0$$

We call this formula the NET PRESENT VALUE (NPV) of the investment and we expect it to be positive in order to make the investment interesting over the time horizon considered.

Outline

1. Introduction
2. Basics
3. Cost-Benefit Analysis
   - Net Present Value
   - Cash Flow Example
   - Net Present Social Value
   - An Example
4. Multi-attribute Value Functions
5. Further Reading

1st example-1

Consider an investment consisting in buying some new machinery (estimated cost 100000 €). The time horizon is fixed to three years with annual operating costs 10000 €. At a fix discount rate of 5%, what should be the annual income (consider it fix every year) in order to make the investment interesting?

1st example-2

Let’s write down the cash flow over the three years

$$\frac{X - 110000}{1.05} + \frac{X - 10000}{1.05^2} + \frac{X - 10000}{1.05^3}$$

where X is the unknown annual income

1st example-3

Putting the cash flow non negative and resolving for X we get: $X \geq 45000$ approximately.
This means that we have to generate approximately a constant annual income of 45000 € in order to be the investment interesting.

2nd example-1

Consider now the same investment and the same operating costs as with the previous example. However, you know now that the first year you can expect an income of 5000 €, the second an income of 45000 € and the third an income of 75000 €. At what discount rate this investment will be interesting (always in a three years horizon)?

2nd example-2

Let's write down the cash flow over the three years

$$\frac{-95000}{1+r} + \frac{35000}{(1+r)^2} + \frac{65000}{(1+r)^3}$$

where r is the unknown discount rate

2nd example-3

Putting the cash flow non negative and resolving for $X = 1 + r$ we get:
$X \geq 1.03$ approximately.
This means that for a discount rate of approximately 3% this investment becomes interesting.

The reader will note that in order to solve the cash flow equation it is necessary to solve a non linear equation (in this case quadratic). In order to do so he should remind to consider only the positive solutions.

Outline

Multiple Costs and Benefits

Consider a project part of a public policy. First hypothesis: *there are multiple and qualitatively different costs and benefits instead of single ones*.

$$B_i(K) = \sum_j b_{ij}(K)$$

$$C_i(K) = \sum_j c_{ij}(K)$$

where $b_{ij}(K)$ is the j^{th} benefit of project K at time i and $c_{ij}(K)$ is the j^{th} cost of project K at time i

Multiple Costs and Benefits

Consider a project part of a public policy. Second hypothesis: *each cost and each benefit should be commensurable, possibly in monetary terms*.

There are two ways to obtain that:

- either there is a market (direct or proxy) where these costs and benefits can be priced;
- or there exist suitable trade-offs between each cost and benefit with a reference (cost or benefit) expressed in monetary terms.

A small example: a highway project

COSTS	BENEFITS
Construction	Accessibility
Maintenance	Time reduction
Landscape	Area development
Pollution	Less accidents
Yard disturbances	Workforce employment

A small example: a highway project

Time reduction

In order to calculate the monetary equivalent of time reduction we can consider the value of time resulting from the job market.

Landscape

In order to calculate the monetary cost of Landscape we can consider the extra construction cost required to avoid each specific landscape deterioration the highway may create (trade-off).

Net Present Social Value

At this point we can calculate the Net Present Social Value of project K.

$$NPSV(K) = \sum_{i=1}^{T} \frac{\overline{b}_i(K) - \overline{c}_i(K)}{(1+r)^i} = \frac{\sum_k h_k b_{ik}(K) - \sum_j p_j c_{ij}(K)}{(1+r)^i}$$

where h_k and p_j represent the trade-offs among the different costs and benefits.

Further implicit hypotheses we did

1. The society is seen as a collection of consumers of goods affected by the project realisation.
2. Any cost and benefit have a price (there is a direct or proxy market where this is fixed).
3. Cost and benefits can compensate one the other.
4. Further generations will still value the projet as we do today (in case the project time horizon spans over several generations).
5. There is no uncertainty as far as the outcomes of the project are concerned.

Procedure Summary

1. Identify a set of potential costs of the project.
2. Identify a set of potential benefits of the project.
3. Establish appropriate prices for each cost and for each benefit.
4. Establish appropriate trade-offs among the different costs and benefits.
5. Fix an appropriate time horizon within which the project should be evaluated as well as the time periods discretising the time horizon.
6. Choose an appropriate discount rate homogenising the future costs and benefits to the present prices.

Results

1. If $NPSV(K) > 0$ then project K is socially profitable. If several projects compete then their $NPSV$ could be used to rank them.
2. The ratio $\frac{B(K)}{C(K)}$ (where $B(K)$ is the overall discounted benefits and $C(K)$ the overall discounted costs represents the project effectiveness. If it superior to 1 then the project is socially effective. If several projects compete this ration could rank them.
3. **Rankings according to $NPSV$ and according to effectiveness may be different.**
4. Solving for $NPSV(K) = 0$ with T unknown establishes the payback period.
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Outline

Highway project

The following example is borrowed from the EU Manual of Cost-Benefit Analysis, see reference in "Further Readings".

The project consists in constructing a new motorway by-passing a densely populated area in order to decrease traffic congestion and air pollution, besides improving accessibility and safety. Two options are considered, a free motorway and a tolled one. It is not expected to observe major increases in traffic, since the area is already heavily developed. It is rather expected to observe traffic diversion, moving from the present local network to the new motorway.

Highway project: hypotheses

- The length of the new motorway si 72km.
- The technical life is 70 years and thus the assessment time horizon has been fixed at 30 years (approximately 40%). The discretised time line has been established in years.
- The social discount rate has been fixed at 5.5%.
- Traffic forecast has been established using conventional traffic and transportation models.

Highway project: COSTS and BENEFITS

COSTS	BENEFITS
Investments	Consumer's Surplus
Works	Time reduction
Land	Vehicle Operating Costs Reduction
Junctions	Gross Producer and Road User Surplus
General	Tolls (in case)
Operating	Vehicle Operating Costs
Maintenance	State Revenues
Other	Environmental Benefits
	Accident Reduction

Free Highway, 0-15 years

	CF	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
BENEFITS																
Consumer's surplus		**0.0**	**0.0**	**0.0**	**0.0**	**53.7**	**56.1**	**58.4**	**60.8**	**63.2**	**65.6**	**68.0**	**70.3**	**72.7**	**75.1**	**77.5**
Time Benefits		0.0	0.0	0.0	0.0	59.9	62.5	65.0	67.6	70.1	72.6	75.2	77.7	80.3	82.8	85.3
Vehicle Operating Costs (perceived)		0.0	0.0	0.0	0.0	-6.3	-6.4	-6.6	-6.7	-6.9	-7.1	-7.2	-7.4	-7.6	-7.7	-7.9
Gross Producer and Road User Surplus		**0.0**	**0.0**	**0.0**	**0.0**	**-10.3**	**-10.6**	**-10.8**	**-11.1**	**-11.3**	**-11.6**	**-11.8**	**-12.1**	**-12.3**	**-12.6**	**-12.8**
Tolls		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Vehicle Operating Costs (not perceived)		0.0	0.0	0.0	0.0	-10.3	-10.6	-10.8	-11.1	-11.3	-11.6	-11.8	-12.1	-12.3	-12.6	-12.8
Net revenues for the State		**0.0**	**0.0**	**0.0**	**0.0**	**10.3**	**10.5**	**10.8**	**11.0**	**11.3**	**11.6**	**11.8**	**12.1**	**12.3**	**12.6**	**12.8**
Net Environmental Benefits		**0.0**	**0.0**	**0.0**	**0.0**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**
Accident reduction		**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**
TOTAL BENEFITS		**0.0**	**0.0**	**0.0**	**0.0**	**53.4**	**55.8**	**58.2**	**60.6**	**63.0**	**65.4**	**67.7**	**70.1**	**72.5**	**74.9**	**77.3**
COSTS																
Investment Costs																
Works	0.794	77.2	115.7	113.8	91.9											
Junctions	0.794	45.6	45.6	45.6	45.6											
Land acquisition	1.000	14.7	14.2	14.7	14.7											
General Expenses	0.998	10.5	10.5	10.5	10.5											
Other expenses	0.998	4.5	4.5	4.5	4.5											
Total investments costs		*152.5*	*190.5*	*189.1*	*167.2*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*
Operating Costs (motorway operator)																
Maintenance	0.573	0.0	0.0	0.0	0.0	0.7	0.7	0.7	0.7	0.7	0.7	1.5	1.5	1.5	1.5	1.5
General Expenses	0.998	0.0	0.0	0.0	0.0	3.2	3.2	3.3	3.3	3.3	3.3	3.3	3.3	3.4	3.4	3.4
Total operating costs		*0.0*	*0.0*	*0.0*	*0.0*	*3.9*	*3.9*	*4.0*	*4.0*	*4.0*	*4.0*	*4.8*	*4.8*	*4.9*	*4.9*	*4.9*
TOTAL COSTS		**152.5**	**190.5**	**189.1**	**167.2**	**3.9**	**3.9**	**4.0**	**4.0**	**4.0**	**4.0**	**4.8**	**4.8**	**4.9**	**4.9**	**4.9**
NET BENEFITS		**-152.5**	**-191.0**	**-189.1**	**-167.2**	**49.5**	**51.9**	**54.2**	**56.6**	**59.0**	**61.4**	**62.9**	**65.3**	**67.6**	**70.0**	**72.4**

Free Highway, 16-30 years

	CF	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BENEFITS																
Consumer's surplus		**79.8**	**82.2**	**84.6**	**87.0**	**88.9**	**90.9**	**92.9**	**94.9**	**96.9**	**98.9**	**98.9**	**98.9**	**98.9**	**98.9**	**98.9**
Time Benefits		87.9	90.4	93.0	95.5	97.6	99.7	101.7	103.8	105.9	108.0	108.0	108.0	108.0	108.0	108.0
Vehicle Operating Costs (perceived)		-8.1	-8.2	-8.4	-8.5	-8.6	-8.7	-8.8	-8.9	-9.0	-9.1	-9.1	-9.1	-9.1	-9.1	-9.1
Gross Producer and Road User Surplus		**-13.1**	**-13.3**	**-13.6**	**-13.8**	**-14.0**	**-14.1**	**-14.3**	**-14.4**	**-14.6**	**-14.7**	**-14.7**	**-14.7**	**-14.7**	**-14.7**	**-14.7**
Tolls		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Vehicle Operating Costs (not perceived)		-13.1	-13.3	-13.6	-13.8	-14.0	-14.1	-14.3	-14.4	-14.6	-14.7	-14.7	-14.7	-14.7	-14.7	-14.7
Net revenues for the State		**13.1**	**13.3**	**13.6**	**13.8**	**14.0**	**14.1**	**14.3**	**14.4**	**14.5**	**14.7**	**14.7**	**14.7**	**14.7**	**14.7**	**14.7**
Net Environmental Benefits		**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**	**-0.3**
Accident reduction		**0.1**	**0.1**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**	**0.2**
TOTAL BENEFITS		-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	0.0	0.0	0.0	0.0	0.0	0.0
BENEFITS		79.7	82.1	84.5	86.8	88.8	90.8	92.8	94.8	96.8	98.8	98.8	98.8	98.8	98.8	98.8
COSTS																
Investment Costs																
Works	0.794															
Junctions	0.794															
Land acquisition	1.000															
General Expenses	0.998															
Other expenses	0.998															
Total investments costs		*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*0.0*	*-279.9*
Operating Costs (motorway operator)																
Maintenance	0.573	1.5	1.5	1.5	2.2	2.2	2.2	3.3	3.3	4.0	4.0	4.0	4.0	4.0	4.0	4.0
General Expenses	0.998	3.4	3.4	3.4	4.3	4.3	4.3	4.3	4.3	4.6	4.6	4.6	4.6	4.6	4.6	4.6
Total operating costs		*4.9*	*4.9*	*4.9*	*6.5*	*6.5*	*6.5*	*7.6*	*7.6*	*8.6*	*8.6*	*8.6*	*8.6*	*8.6*	*8.6*	*8.6*
TOTAL COSTS		4.9	4.9	4.9	6.5	6.5	6.5	7.6	7.6	8.6	8.6	8.6	8.6	8.6	8.6	-271.3
NET BENEFITS		74.7	77.1	79.5	80.3	82.3	84.3	85.1	87.1	88.3	90.2	90.2	90.2	90.2	90.2	370.1

Tolled Highway, 0-15 years

	CF	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
BENEFITS																
Consumer's surplus		**0.0**	**0.0**	**0.0**	**0.0**	**13.4**	**14.7**	**16.1**	**17.5**	**18.8**	**20.2**	**21.5**	**22.9**	**24.2**	**25.6**	**26.9**
Time Benefits		0.0	0.0	0.0	0.0	37.1	38.7	40.3	42.0	43.6	45.2	46.8	48.5	50.1	51.7	53.3
Vehicle Operating Costs (perceived)		0.0	0.0	0.0	0.0	-23.7	-24.0	-24.2	-24.5	-24.8	-25.0	-25.3	-25.6	-25.9	-26.1	-26.4
Gross Producer and Road User Surplus		**0.0**	**0.0**	**0.0**	**0.0**	**23.8**	**24.0**	**24.2**	**24.4**	**24.6**	**24.8**	**25.0**	**25.2**	**25.4**	**25.6**	**25.8**
Tolls		0.0	0.0	0.0	0.0	28.4	28.8	29.1	29.5	29.8	30.2	30.6	30.9	31.3	31.6	32.0
Vehicle Operating Costs (not perceived)		0.0	0.0	0.0	0.0	-4.7	-4.8	-4.9	-5.1	-5.2	-5.4	-5.5	-5.7	-5.8	-6.0	-6.1
Net revenues for the State		**0.0**	**0.0**	**0.0**	**0.0**	**2.4**	**2.5**	**2.6**	**2.6**	**2.7**	**2.8**	**2.9**	**3.0**	**3.1**	**3.1**	**3.2**
Net Environmental Benefits		**0.0**	**0.0**	**0.0**	**0.0**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.1**	**-0.1**
Accident reduction		**0.0**	**0.0**	**0.0**	**0.0**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**	**0.1**
TOTAL BENEFITS		0.0	0.0	0.0	0.0	39.5	41.2	42.8	44.5	46.1	47.8	49.4	51.1	52.8	54.4	56.1
COSTS																
Investment Costs																
Works	0.794	87.3	120.7	129.4	95.3											
Junctions	0.794	45.6	45.6	45.6	45.6											
Land acquisition	1.000	14.7	14.2	14.7	14.7											
General Expenses	0.998	10.5	10.5	10.5	10.5											
Other expenses	0.998	4.5	4.5	4.5	4.5											
Total investments costs		162.6	195.5	204.7	170.6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Operating Costs (motorway operator)																
Maintenance	0.573	0.0	0.0	0.0	0.0	0.7	0.7	0.7	0.7	0.7	0.7	1.5	1.5	1.5	1.5	1.5
General Expenses	0.998	0.0	0.0	0.0	0.0	3.3	3.3	3.4	3.4	3.4	3.4	3.4	3.4	3.4	3.5	3.5
Total operating costs		0.0	0.0	0.0	0.0	4.0	4.0	4.1	4.1	4.1	4.1	4.9	4.9	4.9	5.0	5.0
TOTAL COSTS		162.6	195.5	204.7	170.6	4.0	4.0	4.1	4.1	4.1	4.1	4.9	4.9	4.9	5.0	5.0
NET BENEFITS		-162.6	-196.0	-204.7	-170.6	35.5	37.2	38.8	40.4	42.1	43.7	44.5	46.1	47.8	49.4	51.0

Tolled Highway, 16-30 years

	CF	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
BENEFITS																
Consumer's surplus		28.3	29.6	31.0	32.3	33.6	34.8	36.1	37.3	38.6	39.8	39.8	39.8	39.8	39.8	39.8
Time Benefits		54.9	56.6	58.2	59.8	59.8	62.7	64.1	65.5	66.9	68.4	68.4	68.4	68.4	68.4	68.4
Vehicle Operating Costs (perceived)		-26.7	-26.9	-27.2	-27.5	-27.7	-27.8	-28.0	-28.2	-28.4	-28.6	-28.6	-28.6	-28.6	-28.6	-28.6
Gross Producer and Road User Surplus		26.1	26.3	26.5	26.7	26.8	27.0	27.1	27.3	27.4	27.6	27.6	27.6	27.6	27.6	27.6
Tolls		32.3	32.7	33.0	33.4	33.6	33.8	34.0	34.3	34.5	34.7	34.7	34.7	34.7	34.7	34.7
Vehicle Operating Costs (not perceived)		-6.2	-6.4	-6.5	-6.7	-6.8	-6.8	-6.9	-7.0	-7.1	-7.1	-7.1	-7.1	-7.1	-7.1	-7.1
Net revenues for the State		3.3	3.4	3.5	3.6	3.6	3.7	3.7	3.7	3.8	3.8	3.8	3.8	3.8	3.8	3.8
Net Environmental Benefits		-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
Accident reduction		0.1	0.1	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2	0.2
TOTAL BENEFITS		57.7	59.4	61.0	62.7	64.1	65.6	67.0	68.5	69.9	71.4	71.4	71.4	71.4	71.4	71.4
COSTS																
Investment Costs																
Works	0.794															
Junctions	0.794															
Land acquisition	1.000															
General Expenses	0.998															
Other expenses	0.998															
Total investments costs		0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	-293.5
Operating Costs (motorway operator)																
Maintenance	0.573	1.5	1.5	1.5	2.2	2.2	2.2	3.3	3.3	4.0	4.0	4.0	4.0	4.0	4.0	4.0
General Expenses	0.998	3.5	3.5	3.5	4.4	4.4	4.4	4.4	4.4	4.7	4.7	4.7	4.7	4.7	4.7	4.7
Total operating costs		5.0	5.0	5.0	6.6	6.6	6.6	7.7	7.7	8.7	8.7	8.7	8.7	8.7	8.7	8.7
TOTAL COSTS		5.0	5.0	5.0	6.6	6.6	6.6	7.7	7.7	8.7	8.7	8.7	8.7	8.7	8.7	-284.8
NET BENEFITS		52.7	54.3	56.0	56.0	57.5	58.9	59.3	60.7	61.3	62.7	62.7	62.7	62.7	62.7	356.2

Results

	FREE	TOLLED
NPSV	212.9M€	-41.3€
Return Rate	7.8	5.0
Effectiveness	1.3	0.9

It is clear that only the Free Motorway is socially profitable.

Outline

What about?

Consider a regional plan which is expected to affect the economy, the landscape, the environment and quality of life of citizens.

In order to choose among competing projects we need to compare the consequences that these may have against several different dimensions and identify the “best” ones.

This is not straightforward as it may appear

First Questions

What does it mean better?

- Better for whom?
- How do we measure better on landscape esthetics?
- How do we compare better on landscape esthetics with better on costs?

Arguing CBA

Cost-Benefit Analysis claims that there is a “better” for the society as a whole and this is established computing the consumers’ surplus for each project. However, this implies that consumers’ all have the same preferences and that for all possible consequences there exist markets (direct or proxy) allowing the consumers’ to express such preferences.

It is reasonable to argue both such hypotheses. Consumers/Citizens have conflicting preferences (opinions) on many issues and is unlike that any observable externality has an associable market revealing the preferences.

The Value Functions Hypotheses

Collective Rationality

It makes no sense to try to fix society's preferences (Arrow's theorem). Instead we can look to model a specific decision maker/stakeholder preferences since she could have consistent values.

Subjective Values

If this is true then we can try to "measure" the consequences of any project or policy against such values: this is a subjective value function.

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If this is true then we can try to "measure" the consequences of any project or policy against such values: this is a subjective value function.

Outline

How do we measure better?

Let’s go more formal.

- Let $x, y, z \ldots$ be competing projects within set A;
- Let $d_j(x)$ representing the consequences of project x on dimension d_j;
- Let $d_j(A)$ representing the set of all consequences for all projects in A.

The first step consists in verifying that:

$$\forall j \in D \ \exists \succeq_j \subseteq d_j(A)^2$$

such that $\succeq_j$ is a weak order (consequences should be completely and transitively ordered).

How do we measure better?

If the previous hypothesis is verified then

$$\forall j \in D \;\; \exists h_j : A \mapsto \mathbb{R} : d_j(x) \succeq d_j(y) \Leftrightarrow h_j(x) \geq h_j(y)$$

In other terms for each dimension we can establish a real valued function respecting the decision maker's preferences.

This function is ONLY an ordinal measure of the preferences

Example-1

Suppose you have 4 projects x, y, z, w of urban rehabilitation and an assessment dimension named “esthetics”. You have:
- $d_e(x) =$ statue;
- $d_e(y) =$ fountain;
- $d_e(z) =$ garden;
- $d_e(w) =$ kid’s area;

Preferences expressed could be for instance:
$d_e(x) \succ d_e(y) \succ d_e(z) \sim d_e(w)$
A possible numerical representation could thus be:
$h_e(x) = 3$, $h_e(y) = 2$, $h_e(z) = h_e(w) = 1$

Example-2

Suppose you have 4 projects x, y, z, w of urban rehabilitation and an assessment dimension named “land use”. You have:
- $d_l(x) = 100$sqm;
- $d_l(y) = 50$sqm;
- $d_l(z) = 1000$sqm;
- $d_l(w) = 500$sqm;

Preferences expressed could be for instance (suppose the decision maker dislikes land use:
$d_e(y) \succ d_e(x) \succ d_e(w) \sim d_e(z)$
A possible numerical representation could thus be:
$h_e(y) = 4$, $h_e(x) = 3$, $h_e(w) = 2$ $h_e(z) = 1$, but also:
$h_e(y) = 50$, $h_e(x) = 100$, $h_e(w) = 500$ $h_e(z) = 1000$

Is this sufficient?

For the time being we have the following table:

	d_1-h_1	d_2-h_2	$\ldots$	d_n-h_n
x				
y				
z				
w				
$\vdots$				

The consequences of each action and the numerical representation of the decision maker's preferences (ordinal).

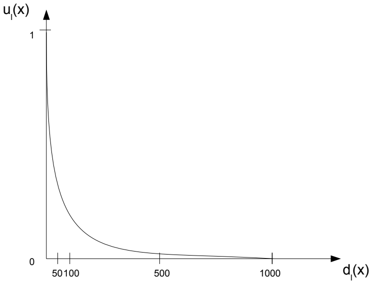
Is this sufficient?

NO!

We need something more rich. We need to know, when we compare x to y (and we prefer x) if this preference is “stronger” to the one expressed when comparing (on the same dimension) z fo w.

We need to compare differences of preferences

An example



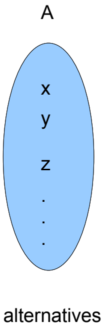
For instance, if the above function represents the value of “land use” it is clear that the difference between 50sqm and 100sqm is far more important from the one between 500sqm and 1000sqm.

First Summary

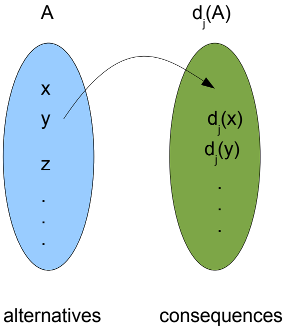
Let's summarise our process until now.

- We get the alternatives.
- We identify their consequences for all relevant dimensions.
- These consequences are ordered for each dimension using the decision maker's preferences.
- We compute the value function measuring the differences of preferences (for each dimension).

First Summary



First Summary



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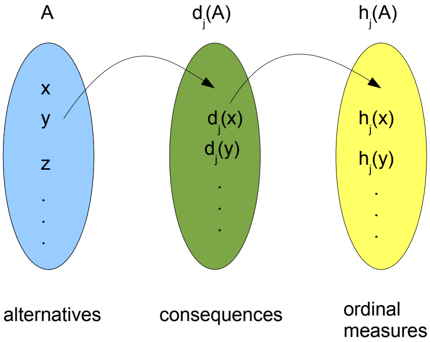

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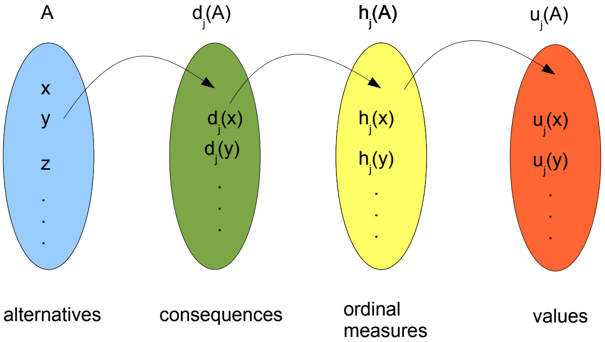

Outline

Is all that sufficient?

NO!

1. The problem is that we need to be able to compare the differences of preferences on one dimension to the differences of preferences on another one (let's say differences of preferences on land use with differences of preferences on esthetics.
2. At the same time we need to take into account the intuitive idea that for a given decision maker certain dimensions are more "important" than other ones.

Principal Hypotheses

1. The different dimensions are separable.
2. Preferences on each dimension are independent.
3. Preferences on each dimension are measurable in terms of differences.
4. Good values on one dimension can compensate bad values on another dimension.

Principal Hypotheses

Under the previous hypotheses we can construct a global value function $U(x)$ as follows:

$$U(x) = \sum_j u_j(x)$$

and in case we use normalised (in the interval [0,1]) marginal value functions $\bar{u}_j$ then:

$$U(x) = \sum_j w_j \bar{u}_j(x)$$

Principal Hypotheses

where: w_j should represent the importance of the marginal functions;

If $h_j(x)$ represent the ordinal values of dimension j then $u_j(d_j(\underline{x})) = 0$ where $d_j(\underline{x})$ is the worst value of h_j

and in case we use normalised value functions then $u_j(d_j(\overline{x})) = 1$ where $d_j(\overline{x})$ is the best value of h_j.

Standard Protocol

1. Fix arbitrary one dimension as the reference for which the value function will be linear (there is no loss of generality doing so).
2. Fix a number of units diving entirely the reference value function, thus fixing the unit of value U_1.
3. Use indifference questions (see later) in order to find equivalent values for the other dimensions.
4. The segments between the equivalent values will shape the other value functions.
5. The ratio of units used to describe each value function with respect to the units for the reference one establishes the trade-offs among the dimensions.

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5. The ratio of units used to describe each value function with respect to the units for the reference one establishes the trade-offs among the dimensions.

Standard Protocol

1. Fix arbitrary one dimension as the reference for which the value function will be linear (there is no loss of generality doing so).
2. Fix a number of units diving entirely the reference value function, thus fixing the unit of value U_1.
3. Use indifference questions (see later) in order to find equivalent values for the other dimensions.
4. The segments between the equivalent values will shape the other value functions.
5. The ratio of units used to describe each value function with respect to the units for the reference one establishes the trade-offs among the dimensions.

Indifference Questions

Given d_r as the reference dimension, h_r being the ordinal preferences we want to establish a value function for dimension d_k. Consider a fictitious object x for which we have $\langle h_r(x), h_k(x) \rangle$. The key question is:

$$\langle h_r(x), h_k(x) \rangle \sim \langle h_r(\bar{x}), ? \rangle$$

What should be the measure on dimension k of an object $\bar{x}$ whose measure on the reference dimension r is such that the $u_r(\bar{x}) = u_r(x) + U_1$ if x and $\bar{x}$ should be indifferent for the decision maker?

Indifference Questions

Once you get the answer $h_k(\bar{x})$ from the decision maker you go ahead:

$$\langle h_r(x), h_k(\bar{x})\rangle \sim \langle h_r(\bar{x}), ?\rangle \quad \rightarrow h_k(\bar{\bar{x}})$$

$$\langle h_r(x), h_k(\bar{\bar{x}})\rangle \sim \langle h_r(\bar{x}), ?\rangle \quad \rightarrow h_k(\bar{\bar{\bar{x}}})$$

Until the whole set of measures of dimension k has been used.

TIPS

TIP1 Start considering a point x at the middle of both scales h_r and h_k.

TIP2 Then start deteriorating on the reference dimension by one unit of value at time (thus the dimension under construction has to improve) until the upper scale of h_k is exhausted.

TIP2 Then start improving on the reference dimension by one unit of value at time (thus the dimension under construction has to deteriorate) until the lower scale of h_k is exhausted.

What do we get?

We have $U(x) = u_r(x) + u_k(x)$ by definition.
We also have $U(\bar{x}) = u_r(\bar{x}) + u_k(\bar{x})$ after questioning.
And since x and $\bar{x}$ are considered indifferent $U(x) = U(\bar{x})$.
Then we get $u_r(x) + u_k(x) = u_r(x) + U_1 + u_k(\bar{x})$ by construction.
We obtain $u_k(\bar{x}) = u_k(x) - U_1$.

Going ahead recursively we found the point $\underline{x}$ at the bottom of the scale for which by definition $u_k(\underline{x}) = 0$ (by definition). Using linear segments between all the points discovered we shape the value function u_k.

Outline

Example

You have to choose among competitive projects assessed against 3 attributes: cost, esthetics and mass. As far as the cost is concerned the scale goes from 5M€ to 10M€. Esthetics are assessed on a subjective scale going from 0 to 8. Mass is measured in kg and the scale goes from 1kg to 5kg. In this precise moment you have under evaluation the following four ones:

project	c	e	m
A	6,5M€	3	3kg
B	7,5M€	4	4,5kg
C	8M€	6	2kg
D	9M€	7	1,5kg

Which is the “best choice”?

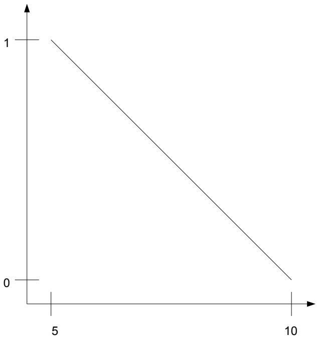
Preferences

First we need to establish appropriate preferences. Suppose in your case the following ones:

- you prefer the less expensive to the more expensive (cost);
- you prefer “pretty” to “less pretty” (esthetics);
- you prefer “heavy” to “less heavy” (mass).

Cost Value Function

Without loss of generality we establish the cost as reference criterion with a linear value function such that $u_c(5\text{M€}) = 1$ and $u_c(10\text{M€}) = 0$. We fix the value unit $U_1 = 0{,}5\text{M€}$.



Cost Value Function

Esthetics Value Function

In order to construct the value function of Esthetics we proceed with the following dialog:

$$\langle 7.5\text{M€}, 4\rangle \sim \langle 8\text{M€}, ?\rangle$$

Consider a project which costs 7.5 € and is assessed on esthetics with 4, and a project which costs 8M € (one unit of value less in this case), how much should the second project be improved in esthetics in order to be indifferent to the first one?
Suppose we get an answer of 5: $\langle 7.5\text{M€}, 4\rangle \sim \langle 8\text{M€}, 5\rangle$
We repeat now the question using the new value:

$$\langle 7.5\text{M€}, 5\rangle \sim \langle 8\text{M€}, ?\rangle$$

We now get an answer of 6.

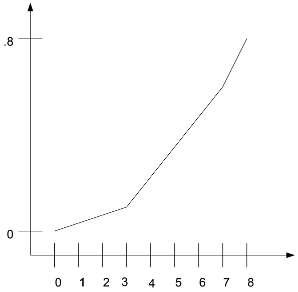
Esthetics Indifferences

We can summarise the dialog as follows:

$$\langle 7.5M€, 4\rangle \sim \langle 8M€, 5\rangle$$
$$\langle 7.5M€, 5\rangle \sim \langle 8M€, 6\rangle$$
$$\langle 7.5M€, 6\rangle \sim \langle 8M€, 7\rangle$$
$$\langle 7.5M€, 7\rangle \sim \langle 8M€, 7.5\rangle$$
$$\langle 7.5M€, 7.5\rangle \sim \langle 8M€, 8\rangle$$
$$\langle 7.5M€, 4\rangle \sim \langle 7M€, 3\rangle$$
$$\langle 7.5M€, 3\rangle \sim \langle 7M€, 1.5\rangle$$
$$\langle 7.5M€, 1.5\rangle \sim \langle 7M€, 0\rangle$$

Esthetics Value Function

The previous dialog will result in the following value function.



Esthetics Value Function

Mass Value Function

In order to construct the value function of Mass we proceed with the following dialog:

$$\langle 7.5\text{M}€, 3.1\rangle \sim \langle 8\text{M}€, ?\rangle$$

Consider a project which costs 7.5€ and weighs 3.1kg and a project which costs 8M€ (one unit of value less in this case), how much should the second project be improved in mass in order to be indifferent to the first one? Suppose we get an answer of 3.5kg: $\langle 7.5\text{M}€, 3.1\rangle \sim \langle 8\text{M}€, 3.5\rangle$
We repeat now the question using the new value:

$$\langle 7.5\text{M}€, 5\rangle \sim \langle 8\text{M}€, ?\rangle$$

We now get an answer of 3.9.

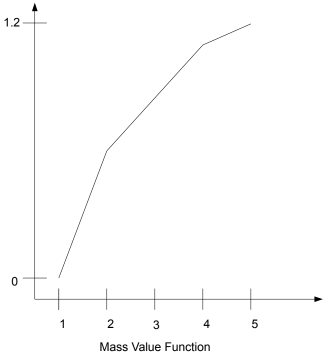
Mass Indifferences

We can summarise the dialog as follows:

$$\langle 7.5\text{M€}, 3.1\rangle \sim \langle 8\text{M€}, 3.5\rangle$$
$$\langle 7.5\text{M€}, 3.5\rangle \sim \langle 8\text{M€}, 3.9\rangle$$
$$\langle 7.5\text{M€}, 3.9\rangle \sim \langle 8\text{M€}, 5\rangle$$
$$\langle 7.5\text{M€}, 3.1\rangle \sim \langle 7\text{M€}, 2.7\rangle$$
$$\langle 7.5\text{M€}, 2.7\rangle \sim \langle 7\text{M€}, 2.3\rangle$$
$$\langle 7.5\text{M€}, 2.3\rangle \sim \langle 7\text{M€}, 1.9\rangle$$
$$\langle 7.5\text{M€}, 1.9\rangle \sim \langle 7\text{M€}, 1.75\rangle$$
$$\langle 7.5\text{M€}, 1.75\rangle \sim \langle 7\text{M€}, 1.6\rangle$$
$$\langle 7.5\text{M€}, 1.6\rangle \sim \langle 7\text{M€}, 1.45\rangle$$
$$\langle 7.5\text{M€}, 1.45\rangle \sim \langle 7\text{M€}, 1.3\rangle$$
$$\langle 7.5\text{M€}, 1.3\rangle \sim \langle 7\text{M€}, 1.15\rangle$$
$$\langle 7.5\text{M€}, 1.15\rangle \sim \langle 7\text{M€}, 1\rangle$$

Mass Value Function

The previous dialog will result in the following value function.



Final calculations

Having obtained the three value functions we can now calculate the values of the four projects for each of them.

$$
\begin{array}{lll}
u_c(A) = 0.7 & u_e(A) = 0.2 & u_m(A) = 0.875 \\
u_c(B) = 0.5 & u_e(B) = 0.3 & u_m(B) = 1.160 \\
u_c(C) = 0.4 & u_e(C) = 0.5 & u_m(C) = 0.625 \\
u_c(D) = 0.2 & u_e(D) = 0.6 & u_m(D) = 0.330
\end{array}
$$

Final Results

Finally we get

$$\begin{aligned}
U_c(A) &= 0.7 + 0.2 + 0.875 = 1.775 \\
U_c(B) &= 0.5 + 0.3 + 1.160 = 1.960 \\
U_c(C) &= 0.4 + 0.5 + 0.625 = 1.525 \\
U_c(D) &= 0.2 + 0.6 + 0.330 = 1.130
\end{aligned}$$

The project which maximises the decision maker's value is B.

Where did the weight disappear?

NOWHERE

Suppose we were using normalised value functions which have to be "weighted". We recall that in such a case we have:

$$U(x) = \sum_j w_j \bar{u}_j(x)$$

Consider the first indifference sentence about esthetics. We had: $\langle 7.5\text{M}€, 4\rangle \sim \langle 8\text{M}€, 5\rangle$. We get:
$w_c\bar{u}_c(7.5\text{M}€) + w_e\bar{u}_e(4) = w_c\bar{u}_c(8\text{M}€) + w_e\bar{u}_e(5)$
where:
- w_c and w_e represent the "weights" of cost and esthetics respectively;
- and $\bar{u}_c$ and $\bar{u}_e$ are the normalised value functions.

Here are the weights ...

By construction $u_c(x) = \bar{u}_c(x)$. We get:
$w_c(\bar{u}_c(7.5\text{M€}) - \bar{u}_c(8\text{M€})) = w_e(\bar{u}_e(5) - \bar{u}_e(4))$. Thus:

$$\frac{w_e}{w_c} = \frac{\bar{u}_c(7.5\text{M€}) - \bar{u}_c(8\text{M€})}{\bar{u}_e(5) - \bar{u}_e(4)}$$

However, $\bar{u}_c(7.5\text{M€}) - \bar{u}_c(8\text{M€}) = 1/10$ of the cost value function (by construction) and $\bar{u}_e(5) - \bar{u}_e(4) = 1/8$ of the esthetics value function as it results from the dialog. Using the same procedure for mass we get:

- $w_e/w_c = 0.8$ meaning that esthetics represents 80% of the cost value (this is the esthetics trade-off);
- $w_m/w_c = 1.2$ meaning that mass represents 120% of the cost value (this is the mass trade-off);

Conclusion and tips

Tip1 Not surprisingly the “weight” of each criterion is represented by the maximum value it attains.

Tip2 It is better not to use any “weights” when constructing value functions, since it can generate confusion to the decision maker. We can explain the relative importance of each criterion using the trade-offs.

So called “weights” are the trade-offs among the value functions and as such are established as soon as the value functions are constructed. They do not exist independently and is not correct to ask the decision maker to express them.

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